CHAPTER 10: ELASTICITY

If a force is applied to a material in such a way as to deform it (change its shape or size), then the material is said to be stressed and there will be change in relative positions of the molecules within the body and the material become strained. Stress which results in increase in length is called tensile stress and one which results in decrease in length is called compressive stress.

Terms used

1. **Elasticity:** This is the ability of the material to regain its original shape and size when the deforming load has been removed.
2. **Elastic material:** This is a material which regains its original shape and size when the deforming load has been removed. E.g. Rubber band, spring.
3. **Elastic deformation:** This is when a material can recover its original length and shape when the deforming load has been removed.
4. **Elastic limit:** This is the maximum load which a material can experience and still regain its original size and shape once the load has been removed.

The elastic limit sometimes coincides with the limit of proportionality.

5. **Proportional limit:** This is the maximum load a material can experience for which the extension created on it is directly proportional to the load applied.
6. **Hooke's law:** it staes that; the extension of a wire or spring is proportional to the applied load provided the proportional limit is not exceeded.

   The law shows that when the molecules of a material are slightly displaced from their mean positions, the restoring force is proportional to its displacement.

I.e. $F \alpha e$ $\boxed{F = ke}$ Where k is the constant of proportionality.

7. **Yield point:** this is a point at which there is a marked increase in extension when the stress or load is increased beyond the elastic limit.

   The internal structure of the material has changed and the crystal planes have effectively slid across each other. At yield poirt the material begin to show plastic behavior.

   Few materials exhibit yield point such as mild steel, brass and bronze.
8. **Plastic deformation:** this is when a material cannot recover its original shape and size when the deforming load has been removed.
9. **Breaking stress/ultimate tensile strength:** it is the maximum stress which can be applied to a material. Or it is the corresponding force per unit area of the narrowest cross section of the wire.
10. **Strength:** this is the ability of a material to with stand an applied force before breaking.

Or it is the maximum force which can be applied to a material without it breaking.

11. Stiffness: this is the ability of a material to resist changing its shape and size.

12. Ductility: it is the ability of the material to be permanently stretched. or it is the ability of the material to be stretched appreciably beyond elastic limit. It can be drawn into different shapes without breaking.

13. Brittleness: it is the ability of the material to break immediately it is stretched beyond to elastic limit.

14. Toughness: this is the ability of material to resist crack growth e.g. rubber

15. Tensile stress: it is force acting per unit area of cross-section of a material.

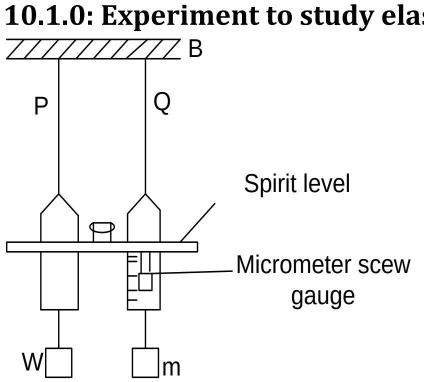
Stress = $\frac{F}{A}$

16. Tensile strain: it is the extension per unit original length of the material.

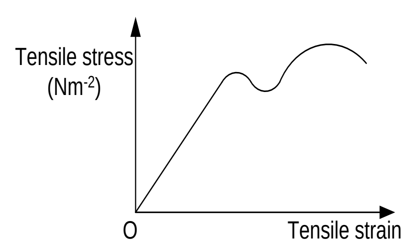
Strain = $\frac{e}{L}$

Strain has no units because it is a ratio of two similar units

10.1.0: Experiment to study elastic properties of steel



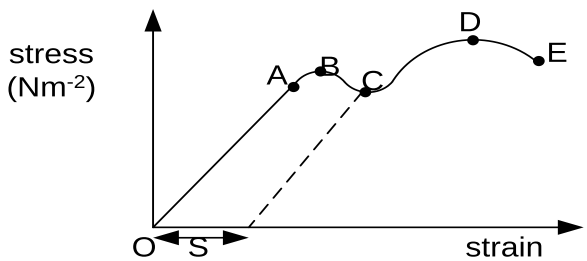
- Two long, thin identical steel wires are suspended besides each other from rigid support B
- The wire P is kept taut and free of kinks by weight A attached to its end
- The original length l of test wire Q is measured and recorded.
- The mean diameter d is determined and cross-sectional area $A = \frac{\pi d^2}{4}$ is found.
- Known weight, W is added to the free end of test wire Q and the corresponding extension e is read form the vernier scale.
- The procedure is repeated for different weights. however, vernier readings are also taken when the loads are removed. It ensures that the elastic limit is not exceeded.
- Results are tabulated including values of tensile stress $\left(\frac{W}{A}\right)$ and tensile strain $\left(\frac{e}{L}\right)$
- The graph of tensile stress versus tensile strain is plotted as below.



10.1.1: Stress-strain graphs

1. **Ductile material** e.g. copper, steel, iron

A ductile material is one which can be permanently stretched



A-Prportional limit

B-Elastic limit

C-Yield ponit

D-Breaking stress

E-Breaking point

Region OA: stress ∝ strain, all extensions are recovered when the load is removed. It is Hooke's law region and Young's Modulus can be defined only in this region.

Region AB: Hooke's law is not obeyed but extension is recovered when load is removed.

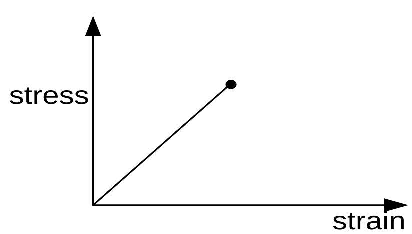
Region BC; Not all extensions are recovered when the load is removed

Region BC: Changes from elastic to plastic deformation

Point E: without any further increase in stress, the wire begins to under go physical changes, it **thins** out at some point and finally breaks

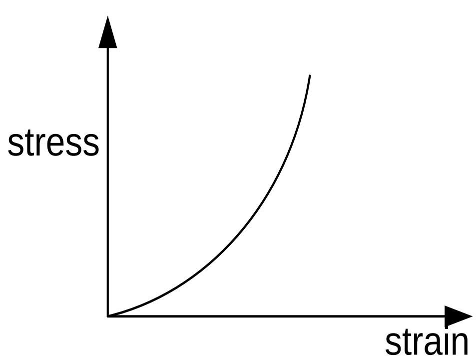
2. **Brittle materials** e.g. glass, chalk, rocks and cast iron

These are materials that can not be permanently stretched. It breaks as soon as the elastic limit has been reached



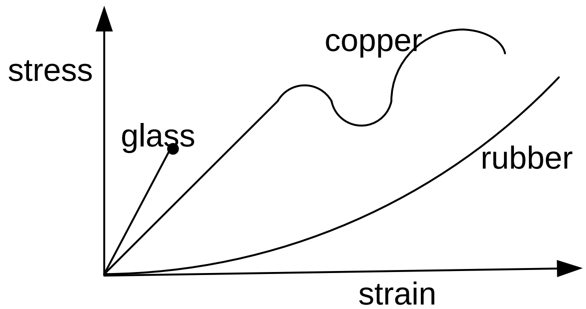
Brittle materials have only a small elastic region and do not under go plastic deformation. This behavior in glass is due to the existence of cracks in its surface. The high concentration of the stress at the crack makes the glass break.

3. **Rubber**



Rubber stretches very easily without breaking and has a greater range of elasticity. Rubber is much less stiff than metals and therefore its value of young's modulus is very much smaller than that of most metals. It does not undergo plastic deformation. Un stretched rubber has coiled molecules and when stretched they unwind and become straight and much harder

10.1.2: Stress-strain graph for glass, copper and rubber



10.1.3: Energy changes/physical process

1. **Elastic deformation**

Particles are slightly displaced from their equilibrium positions and work done by the force to displace the particles is stored as elastic potential energy. When the stretching force is removed, the potential energy of the particles changes to kinetic energy and moves them back to their equilibrium position.

2. **Plastic deformation**

It occurs when the material is stretched beyond the yield point. The crystal planes slide over each other and movement of dislocations takes place. When the stress is removed, original shape and size are not recovered due to energy loss in form of heat.

3. **Work hardening**

It is the process of increasing the resistance of a material to plastic deformation by plastically deforming it repeatedly.

During repeated plastic deformation, the metal dislocations move through the material and they entangle round each other and become immobile, it creates the resistance of the material to plastic deformation.

This explains why it is easier to break a copper wire by flexing it to and fro.

4. **Annealing**

It is a process by which a material restores its ductility.

Procedure

The metal is heated to high temperature above its melting point and maintained in this temperature for a period of time and relaxes the internal strains and hence the metal is re-crystallised and returns to the ductile state.

10.2.0: Young's modulus

It is also called the modulus of elasticity of a wire.

Young's modulus is the ratio of tensile stress to tensile strain of a material

Young modulus = $\frac{stress}{strain}$

$$E = \frac{F/A}{e/L}$$

$$E = \frac{F\,L}{A\,e}$$

A is area, L is original length, e is extension

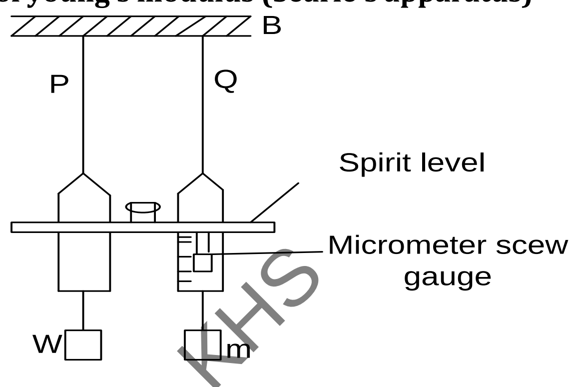
Dimensions of young's modulus

$[E] = \frac{[F][L]}{[A][e]}$

[E] = $\frac{(M\,L\,T^{-2})(L)}{L^2\,L}$

[E]=$M\,L^{-1}T^{-2}$

10.2.1: Determination of young's modulus (Searle's apparatus)



- Two <u>long, thin identical</u> steel wires are suspended besides each other from rigid support B
- The wire P is kept <u>taut</u> and free of <u>kinks</u> by weight A attached to its end
- The <u>original length l</u> of test wire Q is measured and recorded.
- The <u>mean diameter d</u> is determined and cross-sectional area $A = \frac{\pi\,d^2}{4}$ is found.
- Known weight, W is added to the free end of test wire Q and the corresponding extension e is read form the vernier scale.
- The procedure is repeated for different weights. however, vernier readings are also taken when the loads are removed. It ensures that the elastic limit is not exceeded.
- A graph of weight W against extension e is plotted and its slope (s) obtained.
- Young's modulus is obtained from $E = \frac{S\,L}{A}$

Note

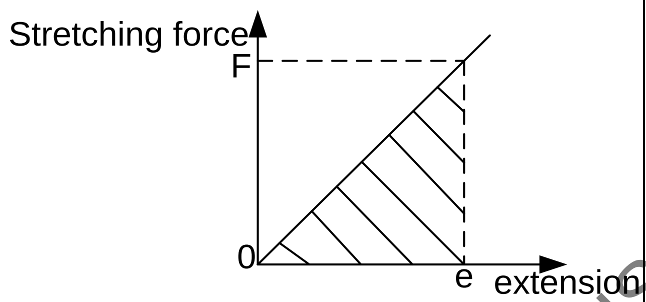
✓ Two identical wires are used to avoid errors due expansion as a result of temperature changes since they are affected equally.

- ✓ Wires are long as it is convenient, because a moderate load would produce a large tensile stress.
- ✓ Wires are thin so that a measurable extension is produced even with a small load. Otherwise if the wires were thick it requires a large load which would cause the support to yield.
- ✓ Micrometer/vernier readings are also taken when the load is removed to ensure that the elastic limit is not exceeded.

10.2.2: Energy stored in a stretched material [strain energy]

Consider a material of an elastic constant k, stretched by a force,$\boldsymbol{F}$ to extend by $\boldsymbol{e}$.

By Hooke's law, the extension is directly proportional to the applied force provided the elastic limit is not exceeded.



Work done = <u>area under the graph</u>

Work done $=\frac{1}{2}F\ e$

But $F = ke$

Work done $=\frac{1}{2}k\ e^2$

The work done to stretch the material is stored as elastic potential in the material

Energy stored $=\frac{1}{2}k\ e^2$

Or Energy stored $=\frac{1}{2}F\ e$

By calculus [integration]

If F is the force which gives an extension from 0 to e and $F = kx$ (from Hooke's law)

Work done $=\int_0^e F\ dx$

$=\int_0^e kx\ dx$

$=\left[\frac{k\,x^2}{2}\right]_0^e$

Work done $=\frac{1}{2}ke^2$

10.2.3: Energy stored per unit volume

Energy stored in the wire $=\frac{1}{2}F\ e$

If a wire is of cross sectional area A and natural length L, the volume = AL

Energy per unit volume $=\frac{Energy\ stored}{volume} = {\frac{1}{2}F\ e}\Big/{AL} = \frac{Fe}{2AL}$

Energy per unit volume $=\frac{1}{2}\left(\frac{F}{A}\frac{e}{L}\right)$ or $\frac{1}{2}x\ stress\ x\ strain$

Numerical examples

1. A metal bar has a circular cross section of diameter 20mm. If the maximum permissible tensile stress is $8x10^7 Nm^{-2}$, calculate the maximum force which the bar can withstand.

Solution

d = 20mm = $20x10^{-3}$m

stress=$\frac{Force}{Area}$

Force = stress x area

$= 8x10^7 x\frac{\pi d^2}{4}$

$= 8x10^7 x\frac{\left[\frac{22}{7}x(20x10^{-3})^2\right]}{4}$

Force = $2.513x10^4$N

2. Find the maximum load which may be placed on steel of diameter 1mm if the permitted strain must not exceed $\frac{1}{1000}$ and young's modulus for steel is $2x10^{11} Nm^{-2}$

Solution

Young modulus $= \frac{stress}{strain}$

Stress = E x strain

$= 2x10^{11} x\frac{1}{1000}$

Stress = $2x10^8 Nm^{-2}$

But stress = $\frac{F}{A}$

Force = stress x area

$= 2x10^8 x\frac{\pi d^2}{4}$

$= 2x10^8 x\frac{\left[\frac{22}{7}x(1x10^{-3})^2\right]}{4}$

Force = $1.571x10^2$N

3. An elastic string of cross-sectional area $4mm^2$ requires a force of 2.8N to increase its length by one tenth. Find young's modulus for the string if the original length of the string was 1m, find the energy stored in the string when it is extended.

Solution

A= $4mm^2$= $4x\ 10^{-6}m^2$,

F = 2.8N,

L=1m, e= $\frac{1}{10}L$ e= 0.1

$E = \frac{F\ L}{A\ e}$

$E = \frac{2.8\ x\ 1}{4x\ 10^{-6}\ x\ 0.1}$

E =$7x10^6 Nm^{-2}$

Energy stored = ½ Fe

$= \frac{1}{2}x2.8x0.1$

= 0.14J

4. A rubber cord of a catapult has a cross-sectional area of $1.2mm^2$ and original length 0.72m, and is stretched to 0.84m to fire a small stone of mass 15g at a bird. Calculate the initial velocity of the stone when it just leaves the catapult. Assume that Young's modulus for rubber is $6.2x10^8\ Nm^{-2}$

Solution

$e = 0.84 - 0.72 = 0.12m$

Stretching force, $F = \frac{EAeL}{l}$

$$F = \frac{6.2x10^8\ x1.2x10^{-6}x0.12}{0.72}$$

$$F = 124N$$

Energy stored in rubber= ½ Fe

$$\frac{1}{2}x124x0.12 = 7.44J$$

Kinetic energy of stone= $\frac{1}{2}mv^2$

$$\frac{1}{2}x0.015xv^2 = 7.44$$

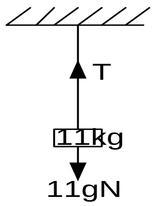
$$v = 31.5ms^{-1}$$

5. A mass of 11kg is suspended from the ceiling by an aluminum wire of length 2m and diameter 2mm, what is;

a) The extension produced

b) The elastic energy stored in the wire (young's modulus of aluminum is 7x10[10]Pa)

Solution



$L = 2m, d = 2mm = 2x10^{-3}m$

$T = 11gN$

$T = 11x9.81$

$T = 107.91N$

$$E = \frac{F\ L}{A\ e}$$

$$e = \frac{F\ L}{A\ E}$$

But $F = T = 107.91N$

$$A = \frac{\pi d^2}{4} = \frac{\left[\frac{22}{7}x(2x10^{-2})^2\right]}{4}$$

$$A = 3.14x10^{-6}m^2$$

$$e = \frac{F\ L}{A\ E}$$

$$e = \frac{107.91x\ 2}{3.14x10^{-6}\ x7x10^{10}}$$

$$e = 9.813x10^{-4}m$$

Energy stored = ½ Te

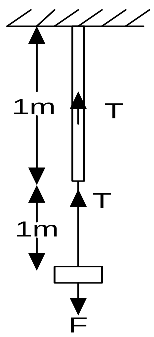
$= ½\ x107.91x9.813x10^{-4}$

Energy stored $= 5.29x10^{-2}$J

6. A cylindrical copper wire and a cylindrical steel wire, each of length 1m and having equal diameter are joined at one end to form a composite wire 2m long. This composite wire is subjected to a tensile stress until its length becomes 2.002m. calculate the tensile stress applied to the wire (young modulus of copper $=1.2x10^{11}$Pa and Steel $=2x10^{11}$Pa)

Solution

[Recall from S.H.M wire in series experience the same tension and weight]



Total extension, $e = 2.002 - 2$

$e = 0.002m$

$e = e_1 + e_2$--------------[1]

Note the two wires will experiences same stress

$0.002 = e_1 + e_2$

But E= $\frac{F\ L}{A\ e}$

$$e = \frac{F\ L}{A\ E}$$

$$0.002 = \frac{F\ L_1}{A\ E_1} + \frac{F\ L_2}{A\ E_2}$$

$$0.002 = \frac{F}{A}\left(\frac{L_1}{E_1} + \frac{L_2}{E_2}\right)$$

$$0.002 = \frac{F}{A}\left(\frac{1}{1.2x10^{11}} + \frac{1}{2x10^{11}}\right)$$

$$\frac{F}{A} = 1.5x10^8N$$

Stress $= 1.5x10^8N$

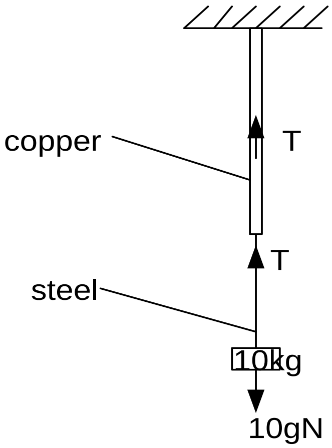
7. One end of a copper wire is welded to a steel wire of length 1.5m and diameter 1mm while the other end is fixed. The length of the copper wire is 0.8m while its diameter is 0.5mm. a bob 10kg is suspended from the free end of a steel wire. Find

i) Extension which results

ii) Energy stored in the compound wire

(Young's modulus for copper$=1x10^{11}$Nm^{-2} and steel $=2x10^{11}$Nm^{-2})

Solution



$E_1 = 1x10^{11}, l_1 = 0.8m$

$d_1 = 0.5mm = 0.5x10^{-3}m$

$E_2 = 2x10^{11}, l_2 = 1.6m$

$d_1 = 1mm = 1x10^{-3}m$

Recall from S.H.M for series wires

T=mg

But $e = \frac{F\,L}{A\,E}$

$e_1 = \frac{F}{A_1} x \frac{L_1}{E_1}$

$$e_1 = \frac{10x9.81x0.8}{\pi \frac{d^2}{4} x1x10^{11}}$$

$$e_1 = \frac{10x9.81x0.8}{\frac{22}{7} x \frac{(0.5x10^{-3})^2}{4} x1x10^{11}}$$

e_1=3.997x10^{-3}m

$e_2 = \frac{F}{A_2} x \frac{L_2}{E_2}$

$$e_2 = \frac{10x9.81x1.6}{\pi \frac{d^2}{4} x2x10^{11}}$$

$$e_2 = \frac{10x9.81x1.6}{\frac{22}{7} x \frac{(1x10^{-3})^2}{4} x2x10^{11}}$$

e_2=9.9924x10^{-4} m

$e = e_1 + e_2$

e=9.9924x10^{-4} + 3.997x10^{-3}

$e = 1.039x10^{-3}m$

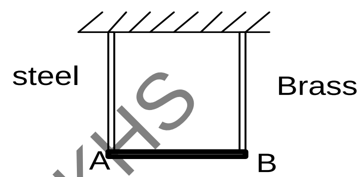
iii) Energy stored in composite

$= \frac{1}{2}$Fe

$= \frac{1}{2} x(10x9.81)x1.039x10^{-3}$

= 5.10x10^{-2}J

7.



A light rigid bar is suspended horizontally from two vertical wires, one of steel and one of brass as shown in the diagram. Each wire is 2.00m long. The diameter of the steel wire is 0.6mm and the length of the bar AB is 0.2m. when a mass of 10kg is suspended from the centre of AB the bar remains horizontal.

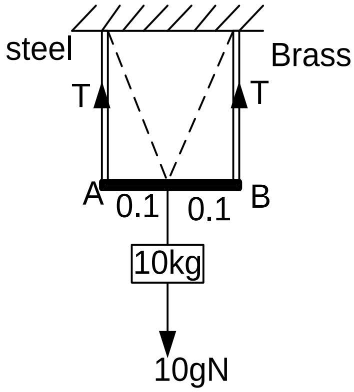
(i) What is the tension in each wire

(ii) Calculate the extension of the steel wire and the energy stored in it

(iii) Calculate the diameter of the brass wire

(iv) If the brass wires are replaced by another brass wire of diameter 1mm, where should the mass be suspended so that AB would remain horizontal.[young's modulus for steel = 2x10^{11}Pa and brass= 1x10^{11}Pa].

Solution



Assume that AB always remains horizontal

L_1 = 2m, d_1 =$0.6x10^{-3}$m, E_1 = $2x10^{11}$pa, e_1 = ?,

L_2 = 2m, d_2 = ?, E_2 = $1x10^{11}$pa, e_2 = ?

Taking moments about O: $0.1xT_1 = 0.1xT_2$

$$T_1 = T_2 \ldots\ldots(i)$$

Also: $10gN = T_1 + T_2 \ldots.(ii)$

$$2T_1 = 10x9.81$$

$$T_1 = 49.05N$$

Tension on each wire is $49.05N$

ii) for steel $e = \frac{F\,L}{A\,E}$

$$e_1 = \frac{T_1}{A_1}x\frac{L_1}{E_1}$$

$$e_1 = \frac{49.05x2}{\frac{22}{7}x\frac{(0.6x10^{-3})^2}{4}x2x10^{11}}$$

e_1 = $1.735x10^{-3}m$

Energy stored in steel = $\frac{1}{2}T_1e_1$

$=\frac{1}{2}x49.05x1.735x10^{-3}$

Energy stored in steel is $4.26x10^{-2}J$

iii) For the bar AB to remain horizontal $e_1 = e_2$ and $L_1 = L_2$

For brass : $A_2 = \frac{T_2}{e_2}x\frac{L_2}{E_2}$

$$A_2 = \frac{49.05}{1.735x10^{-3}}x\frac{2}{1x10^{11}}$$

$A_2 = 5.65x10^{-7}$m^2

$A_2 = \frac{\pi\,d^2}{4}$

$$d^2 = \frac{4x\,5.65x10^{-7}}{\frac{22}{7}}$$

d = $8.485x10^{-4}m$

(v) Brass: d=1mm

$$A_2 = \frac{\pi\,(1x10^{-3})^2}{4}$$

A_2= $7.85x10^{-7}$m^2

$T_1 = \frac{e_1E_1A_1}{L_1}$ and $T_2 = \frac{e_2E_2A_2}{L_2}$

Taking moments about O

$$yxT_1 = (0.2-y)xT_2$$

y($2x10^{11}x2.825x10^{-7}$) =(0.2-y)($1x10^{11}x7.85x10^{-7}$)

$$y = 0.116m$$

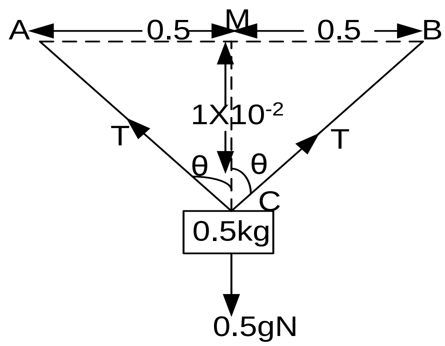
Mass should be placed 0.116m from the steel wire

8. The ends of a uniform wire of cross-sectional area 10^{-6}m^2 and negligible mass are attached to fixed points A and B which are 1m apart in the same horizontal plane. The wire is initially straight and outstretched. A mass of 0.5kg is attached to the mid point of the wire and hangs in equilibrium with the mid point at a distance 10mm below AB. Calculate the value of young's modulus for the wire

Solution

A = 10^{-6}m^2, AB = 1m, e = 1m, m = 0.5kg,

Mc = $10x10^{-3}$m



Using Pythagoras theorem

$CB^2 = 0.5^2 + (1x10^{-2})^2$

$CB^2 = 0.2501$

$CB = 0.5001m$

$AC = CB = 0.5001m$

Length ACB = $0.5001 \, x \, 2$

$= 1.0002m$

Extension = 1.0002 – 1

$e = 2x10^{-4}m$

But $Tan\theta = \frac{0.5}{1x10^{-2}}$

$\theta = 88.9^0$

Resolving vertically

$2Tcos\,\theta = 0.5g$

$2Tcos88.9 = 0.5x9.81$

$T = 127.75N$

$E = \frac{F\,L}{A\,e}$

But F = T (deforming force)

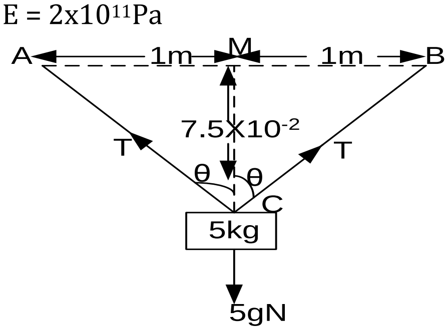
E = $\frac{127.75x1}{10^{-6}x2x10^{-4}}$

E = $6.39x10^{11}Nm^{-2}$

9. The ends of a uniform wire of length 2m are fixed to two points which are 2m apart in the same horizontal line. When a 5kg mass is attached to the mid point of the wire, the equilibrium position is 7.5cm below the line AB. Given that the young's modulus of the material of the wire is $2x10^{11}$pa. find the;
   i. Strain in the wire
   ii. Stress in the wire
   iii. Energy stored in the wire.

Solution

M = 5kg, AB = 2m, L = 2m, $M_C = 7.5x10^{-2}$m,
E = $2x10^{11}$Pa



$CB^2 = MB^2 + MC^2$

$CB^2 = 1^2 + (7.5x10^{-2})^2$

CB = 1,003m

CB = AC = 1.003m

Stretched length ACB = $2x1.003$

= 2.006

Extension = 2.006 – 2= 0.006m

Strain = $\frac{e}{l} = \frac{0.006}{2}$

Strain = $3x10^{-3}$

Stress = $E \, x$ strain= $2x10^{11}$ x 3 x 10^{-3}

Stress = $6x10^8Nm^{-2}$

Energy stored =$\frac{1}{2}Te$ ……………..(i)

But resolving vertically

2Tcosθ = 5g ……………………(ii)

Also $tan\theta = \frac{1}{7.5x10^{-2}}$

$\theta = 85.7^0$

$2Tcos85.7 = 5x9.81$

T = 327.92N

Energy stored = ½ x327.92 x 0.006

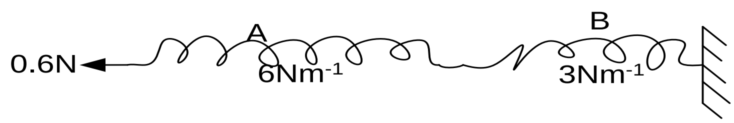
= 9.84×10^{-1}J

Exercise: 25 [use g = 10ms^{-2}]

1. A metal specimen has length of 0.5m. If the maximum permissible strain is not to exceed 10^{-3}, calculate its maximum extension **An (5x10^{-4}m)**

2. A metal bar of length 50mm and square cross-sectional side 20mm is extended by 0.015mm under a tensile load of 30kg, calculate
   a. Stress
   b. Strain in specimen
   c. Value of young's modulus for that metal. **An[7.25x10^{-3}Nm^{-2}, 3x10^{-4}, 24.5Nm^{-2}]**

3.



A spring A of force constant 6Nm^{-1} is connected in series with a spring B of force constant 3Nm^{-1} as shown below. One end of the combination is securely anchored and a force of 0.6N is applied to the other end
   a. By how much does each spring extend
   b. What is the force constant of the combination **An[0.1 (A), 0.2m(B), 2Nm^{-1}]**

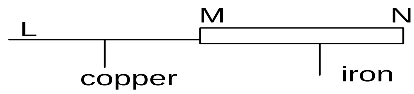
4. A copper wire and steel wire each of length 1.5 m and diameter 2mm are joined end to end to form a composite wire. The composite wire is loaded until its length becomes 3.003m. if young's modulus of steel is $2.0x10^{11}Pa$, and that of copper is $1.2x10^{11}Pa$
   (i) Find the strain in the copper and steel wires
   (ii) Calculate the force applied
   An[copper =0.0013, steel =$7.5x10^{-4}$, force=$4.7x10^{2}N$]

5. A thin steel wire initially 1.5m long and of diameter 0.50mm is suspended from a rigid support, calculate
   i. The final extension
   ii. Energy stored in a wire when a mass of 3kg is attached to the lower end. (young's modulus for steel = 2x10^{11}Nm^{-2}) **An [1.1mm, 1.7x10^{-2}J]**

6. Two wires of steel and phosphor bronze each of diameter 0.40cm and length 3.0m are joined end to end to form a composite wire of length 6.0m. calculate the tension in the wire needed to

produce a total extension of 0.128cm in the composite wire. (Given that E of steel= $2.0x10^{11}Pa$ and E of bronze=$1.2x10^{11}Pa$) **An[100.5N]**

7. A copper wire LM is fused at one end M to an iron wire MN. The copper wire has length 0.9m and cross section $0.9x10^{-6}m^2$. The iron wire has length 1.4m and cross-section $1.3x10^{-6}m^2$. The compound wire is stretched and its total length increases by 0.01m



Calculate;

a) The ratio of the extension of the two wires
b) The extension of each wire
c) The tension applied to the compound wire (young's modulus for copper = $1.3x10^{11}Nm^{-2}$and $2.1x10^{11}$ pa) (Young's modulus for steel = $2x10^{11}Nm^{-2}$)

An (Cu:Fe 3:2, 0.6mm, 4.0mm,780N)

8. a) Define stress, strain and the young's modulus

b) i) describe an experiment to determine the young's modulus for a material in the form of a wire

ii) Which measurement require particular care, from the point of view of accuracy and why

c) i) derive an expression for the potential energy stored in a stretched wire

ii) A steel wire of diameter 1mm and length 1.m is stretched by a force of 50N, calculate the potential energy stored in the wire. ((young's modulus for steel = $2x10^{11}Nm^{-2}$)

An $1.2x10^{-2}$J

iii) The wire is further stretched to breaking where does the stored energy go

9. a) A heavy rigid bar is supported horizontally from a fixed support by two vertical wires A and B of the same initial length and which experience the same extension. If the ratio of the diameter of A and to that of B is 2 and the ratio of the young's modulus of A to that of B is 2, calculate the ratio of the tension in A to that in B. **An (8:1)**

b) if the distance between the wires is D, calculate the distance of wire A from the centre of gravity of the bar**. (An=** $\frac{D}{9}$**)**

10. a) A rubber cord has a diameter of 5.0mm and on un stretched length of 1.0m. One end of the cord is attached to a fixed support A. When a mass of 1.0kg is attached to the other end of the cord so as

to hang vertically below A, the cord is observed to elongate by 100mm, calculate the young's modulus of rubber.

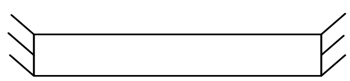
b) If the 1kg mass is now pulled down a further short distance and then released, what is the period of the resulting oscillations? **An [5.1x10^{6}Nm^{-2}, 0.63s]**

11. A uniform steel wire of density 7800kgm^{-3} weighs 26g and 250cm long, it lengthens by 1.2mm, when stretched by a force of 80N, calculate;
    a) The value of young's modulus for steel
    b) The energy stored in the wire
    (**Hint volume = Al =** $\frac{mass}{density}$**)** **Ans (2.03x10^{11}Nm^{-2}, 0.048J**)

10.2.4: FORCE ON A BAR DUE TO THERMAL EXPANSION OR CONTRACTION

When a bar is heated and then prevented from contracting as it cools, a force is exerted at the ends of a bar.

Consider a metal of young's modulus E, cross sectional Area A at a temperature θ_2^{0}c fixed between two rigid supports.



L_2

When the bar is cooled to a temperature θ_1°C, the bar can not contract hence there will be forces on the rigid support.

If α is the mean co-efficient of linear expansion then $L_\theta = L_0(1+\alpha\theta)$

L_θ is length of the bar at temperature θ^{0}c

L_o is length of the bar at temperature 0^{0}c

$L_2 = L_0(1+\alpha\theta_2)$ …………………i

$L_1 = L_0(1+\alpha\theta_1)$ ………………….ii

Subtracting

$L_2 - L_1 = L_0\,\alpha\,(\theta_2 - \theta_1)$

$L_2 - L_1 = L_0\,\alpha\theta$

$\alpha\,\theta = \frac{L_2-L_1}{L_0}$

But strain = $\frac{L_2-L_1}{L_0}$

$\boxed{Strain\ =\ \alpha\,\theta}$ where θ = $\theta_2 - \theta_1$

From E = stress/strain

Stress = E x strain

$\frac{F}{A} = Ex\alpha\,\theta$

F = AE α θ

$\boxed{F\ = AE\ \alpha\ \theta}$

Coefficient of linear expansion α is defined as the fractional increase in length at0°C for every degree rise in temperature.

Examples UNEB 2012 No1c (ii)

1. Two identical steel bars A and B of radius 2.0mm are suspended from the ceiling. A mass of 2.0kg is attached to the free end of bar A, calculate the temperature to which B should be raised so that the bars are again of equal length. (young's modulus of steel = 1.0x10^{11}Nm^{-2} and linear expansitivity of steel = $1.2x10^{-5}K^{-1}$)

Solution

For steel bar A, $r = 2x10^{-3}m$, m = 2kg,

$E = 1x10^{11}Nm^{-2}, \alpha = 1.2x10^{-5}K^{-1}$

But E = $\frac{stress}{strain}$ Strain = $\frac{stress}{E}$

Stress = $\frac{F}{A}$

Strain = $\frac{F}{A\,E}$

Strain = $\frac{2x9.81}{\pi(2x10^{-3})^2x1x10^{11}}$

Strain = $1.56x10^{-5}$

but strain = α θ

θ = $\frac{1.56x10^{-5}}{1.2x10^{-5}}$ θ = 1.3K

B should be raised by a temperature of 1.3K

2. A uniform metal bar of length 1m and diameter 2cm is fixed between two rigid supports at 25^0C. if the temperature of the bar is raised to 75^0C, find

(i) The force exerted on the support.

(ii) Energy stored in the bar at 75^0C. (young's modulus of metal = $2x10^{11}$Pa and coefficient of linear expansion = $1x10^{-5}K^{-1}$)

Solution

i) $\Theta_1 = 25^0C$, $\Theta_2 = 75^0C$, E = $2x10^{11}$Pa,

L = 1m,

d = $2x10^{-2}$m, α = $1x10^{-5}K^{-1}$

Force = EA α θ

= $2x10^{11}\ x\ \frac{\pi\, d^2}{4} x1x10^{-5}(\theta_2 - \theta_1)$

= $2x10^{11}\ x\ \frac{\frac{22}{7}x\,(2x10^{-2})^2}{4} x1x10^{-5}(75 - 25)$

F = $3.14x10^4$N

ii) Energy stored = $\frac{1}{2}$Fe

but strain = α θ

and also strain = $\frac{e}{l}$

$\frac{e}{l}$ = α θ

e = $l\,\alpha\,\theta$

Energy stored = $\frac{1}{2}$F $l\,\alpha\,\theta$

= $3.14x10^4\ x1x10^{-5}x1x(75 - 25)$

= 7.85J

Exercise:26

1. A copper rod of length 0.8m and diameter 40mm is fixed between two rigid supports at a temperature of 20^0C. the temperature of the rod is raised to 70^0C, calculate;

   i. The force exerted on the rod at 70^0C
   ii. Energy stored per unit volume at 70^0C
   iii. Force exerted on the support if temperature was lowered to 45^0C

   [E for copper = $1.2x10^{11}Nm^{-2}$, α for copper between 20^0C to 70^0C is $1.7x10^{-5}K^{-1}$]

   Ans ($1.28x10^5$ N, 43.52J, $4.33x10^4Jm^{-3}$, $6.4x10^4$N)

2. Two identical cylindrical steel bars each of radius 3.00m and length 7m rest in a vertical position with their lower end on a rigid horizontal surface. A mass of 4.0kg is placed on the top of one bar.

The temperature of the other bar is to be altered so that the two bars are once again of equal length. Given that the coefficient of linear expansivity of steel is= $1.2x10^{-5}K^{-1}$

(i) By how much should the temperature be altered

(ii) Find the energy store in the bar due to the temperature change. **An[0.58K, $\mathbf{0.96J}$]**

UNEB 2014 No2

(a) (i) What is meant by **Young's modulus** (01mark)

(ii) State **Hooke's law** (01mark)

(iii) Derive an expression for the energy released in a unit volume of a stretched wire in terms of stress and strain (04marks)

(b) A steel wire of length 0.6 m and cross- sectional area $1.5x10^{-6}m^2$ is attached at B to a copper wire BC of length 0.39 m and cross-sectional area $3.0x10^{-6}m^2$. The combination is suspended vertically from a fixed point at A and supports weight of 250 N at C. find the extension in each of the wires, given that Young's Modulus for steel is $2.0x10^{11}Pa$ and that of copper is $1.3x10^{11}Pa$.

An[$\boldsymbol{steel = 5.0x10^{-4}m, copper = 2.5x10^{-4}m}$] (05 marks)

(c) With the aid of a labeled diagram, describe an experiment to determine the Young's Modulus of steel wire (07marks)

(d) Explain the term plastic deformation in metals (02marks)

UNEB 2012 No1

a) State Hooke's law (I mark)

b) A copper wire is stretched until it breaks

i. Sketch a stress-strain graph for the wire and explain what happens to the energy used to stretch the wire at each stage. (4 marks)

ii. Derive the expression for the work done by a distance e (3 marks)

c) Define young's modulus (1 mark)

UNEB 2010

a) i) describe the terms tensile stress and tensile strain as applied to a stretched wire. (2 marks)

b) ii) Distinguish between elastic limit and proportional limit (2 marks)

c) With the aid of a labeled diagram, describe an experiment to investigate the relationship between tensile stress and tensile strain of a steel wire (4 marks)

d) i) A load of 60N is applied to a steel wire of length 2.5m and cross sectional area of $0.22mm^2$. if young's modulus for steel is 210GPa, find the expansion produced. (3 marks)

ii) If the temperature rise of 1k causes a fractional increase of 0.001%, find the change in the length of a steel wire of length 2.5mm when the temperature increases by 4K. (3 marks)

Solution

F = 60N, L = 2.5m, A = 0.22mm^2 = 0.22x10^{-6}m^2, E = 210Gpa or E = 210x10^{9}Pa

Expansion required is the extension

$$E = \frac{F\,L}{A\,e}$$

$$e = \frac{F\,L}{AE}$$

$$e = \frac{60x2.5}{0.22x210x10^9x10^{-6}}$$

e = 3.247 x 10^{-3}m

ii) 1K gives 0.001%

%extension = $\frac{extension}{natural\ length}x100\%$

0.001% = $\frac{e}{2.5}x100\%$

e = 2.5x10^{-4}m

1K = 2.5x10^{-4}m

4K = 2.5x10^{-4}x4

4K = 1x10^{-3}m

UNEB 2006 No 3

a) i) Define stress and strain (2 marks)

ii) Determine the dimensions of young's modulus (3 marks)

b) Sketch a graph of stress versus strain for a ductile material and explain its features (6 marks)

c) A steel wire of cross-section area 1mm^2 is cooled from a temperature of 60^{0}c to 15^{0}c, find the;

i. Strain (2marks)

ii. Force needed to prevent it from contracting young's modulus = 2x10^{11}Pa, coefficient of linear expansion for steel = 1.1x10^{-5}K^{-1} (3 marks)

d) Explain the energy changes which occur during plastic deformation (4 marks)

Ans (4.95x10^{-4}, 99N)

UNEB 2005 No 2

a) Explain the terms

i. Ductility

ii. Stiffness

b) A copper wire and steel wire each of length 1.0m and diameter 1.0mm are joined and to end to form a composite wire 2.0m long, find the strain in each wire when the composite stretches by 2x10^{-3}m Young's modulus for copper and steel are 1.2 x10^{11} and 2.0x10^{11}Pa respectively

Ans (1.25x10^{-3}, 7.5x10^{-4}) (7 marks)

UNEB 2003 No 3(d)

i) define the terms longitudinal stress and young's modulus of elasticity (2 marks)

ii) describe how to determine young's modulus for a steel wire. (07 marks)

UNEB 2001 No2

a) Define the following terms

i. Stress (1 mark)

ii. Strain (1 mark)

c) State the necessary measurements in the determination of young's modulus of a metal wire (2 marks)

d) Explain why the following precautions are taken during an experiment to determine young's modulus of a metal wire.

i. Two long, thin wires of the same material are suspended from a common support (2 marks)

ii. The readings of the Vanier are also taken when the loads are gradually removed in steps (1 mark)

CHAPTER 11: FLUID MECHANICS

Both liquids and gases are fluids because they 'flow' i.e. since their molecules are spaced and cause a change in shape without change in volume.

Fluid mechanics involves fluids at rest (hydro statistics) and fluids in motion (hydro dynamics/fluid flow)

11.1.0: Fluid flow

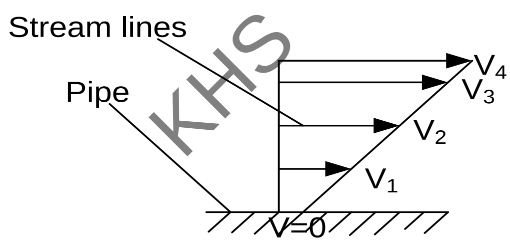
11.1.1: Viscosity

Definition

Viscosity is the frictional force between adjacent layers of a fluid moving at different velocities.

Fluid flow involves a shear. Fluids flow or move in form of layers, adjacent layers of a fluid are displaced over each other to form a shear. The different layers move at different speeds and therefore there will be a frictional force which opposes relative motion between the layers of the fluid. This frictional force is called **viscosity**. The greater the viscosity, the less easily it is for the liquid to flow and the stickier the liquid feels, the harder the liquid to flow.

Fluids stick to the solid surface so that when they flow the velocity must gradually decrease to zero as the wall of the pipe or vessel in approached.



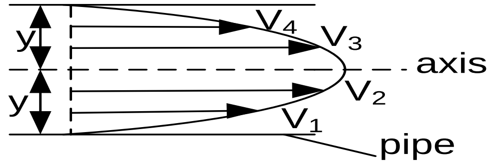
The arrows are known as stream lines and the length of the streamlines represents the magnitude of the velocity.

Definition: A streamline is a curve whose tangent at any point is along the direction of the velocity of the fluid particles at that point.

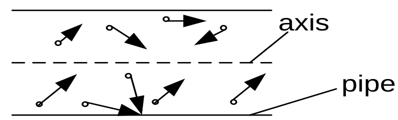
11.1.2: LAMINAR AND TURBULENT FLOW

Laminar (steady/uniform) flow is the orderly flow of a liquid where lines of flow are parallel to the axis of the tube or pipe and liquid particles at the same distance from the axis have the same velocity.

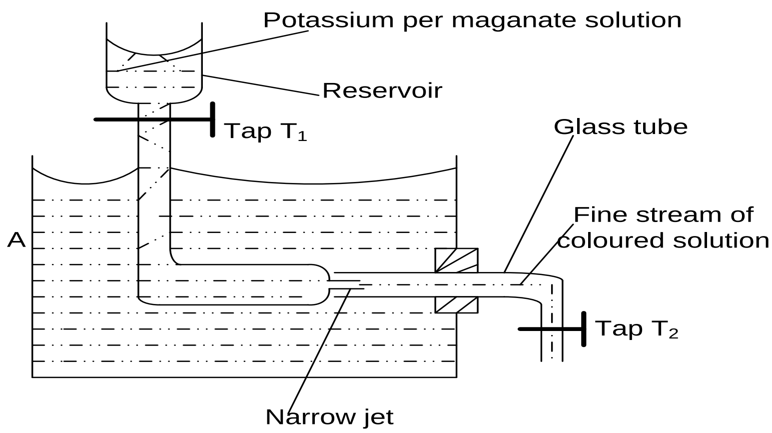
Laminar flow occurs at low velocities below the critical velocity.



Turbulent flow is a disorderly flow where lines of flow are not parallel to the axis of the pipe and liquid particles at the same distance from the axis have different velocities (speeds and direction). Turbulent flow occurs at high velocities, above the critical velocity.

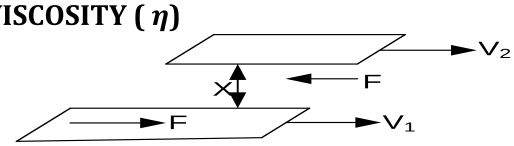


11.1.3: EXPERIMENT TO DEMONSTRATE LAMINAR AND TURBULENT FLOW



- ❖ The apparatus is set up as above. With taps T_1 andT_2 closed, potassium per magnate solution is poured into the reservoir and water poured into container A
- ❖ T_2 is then slightly opened to allow water to flow out of the glass tube and T_1 also opened slightly. A fine stream coloured solution is seen flowing along side the water to the glass tube and this illustrates laminar flow.
- ❖ Tap T_2 is then widely opened to allow more water to flow from the glass tube, a stage is reached when the coloured solution in the glass tube begins to spread out and fill the wall of the tube. The critical velocity has been exceeded and turbulence has begun.

11.1.4: COEFFICIENT OF VISCOSITY (η)



Consider two parallel layers of a liquid moving with velocities V_1 and V_2 and separated by a distance x with area of contact between the layers A

The slower lower layer exerts a tangential retarding force F on the faster upper layer the lower layer its self-experiences an equal and opposite tangential force F due to the upper layer.

Velocity gradient between the layers $= \frac{Velocity\ change}{distance\ apart} = \frac{V_2 - V_1}{x}$

Definition

Velocity gradient is the change in velocity between two layers (points) per unit length of separation of the points.

Frictional force F between adjacent layers depends on

Area of contact between the layers [F α A]

Velocity gradient between layers [F α velocity gradient]

Therefore $F \ \alpha \ A \ x \ velocity \ gradient$

$$F = \eta x \ A \ x \ Velocity \ gradient$$

$$\boldsymbol{\eta} = \frac{F}{A \ x \ Velocity \ gradient}$$

Definition

Coefficient of viscosity is the frictional force acting on a unit area of a fluid when it is in a region of unit velocity gradient

OR

Coefficient of viscosity is the tangential stress which one layer of a fluid exerts on another layer in contact with it when the velocity gradient between the layers is $1s^{-1}$.

Dimensions of $\boldsymbol{\eta}$

$$\boldsymbol{\eta} = \frac{F}{A \ x \ Velocity \ gradient}$$

$$[\boldsymbol{\eta}] = \frac{[F]}{[A]x[Velocity \ gradient]}$$

$$[\boldsymbol{\eta}] = \frac{M \ L \ T^{-1}}{L^2\left(\frac{L \ T^{-1}}{L}\right)}$$

$$[\boldsymbol{\eta}] = \boldsymbol{M \ L^{-1} \ T^{-1}}$$

Units of $\boldsymbol{\eta}$ = $\mathbf{Nsm^{-2}}$

11.1.5: Effects of temperature on viscosity

- In liquids, viscosity is due to intermolecular forces of attraction between layers moving at different speeds. Increase in temperature reduces(weakens) intermolecular forces which increases molecular separation and speed, consequently viscosity in liquids decreases rapidly with increase in temperature
- In gases, viscosity is due to transfer of momentum . Molecules are further apart and have negligible intermolecular forces , molecules move randomly colliding with one another and continuously transferring momentum to the neighboring layers. Increasing the temperature of the gas increases the average speed (increases K.E) of the gas molecules hence increasing the transfer of momentum which results into increase in viscosity of the gas.

Differences between viscosity and solid friction

Solid friction	Viscosity
Independent of area of contact	Depends on area of contact
Independent of relative velocity between layers in contact	Directly proportional to velocity gradient
Independent of temperature but dependent on normal reaction	Depends on temperature

11.1.6: Steady flow of a liquid through a pipe (poiseuille's formula)

Poiseuille derived an expression for the volume of a liquid flowing out of a pipe per second. He assumes that the flow was steady/laminar.

The volume of liquid flowing out of a pipe per unit time (V/t) dependents on;

i. The coefficient of viscosity η of the liquid
ii. The radius of the pipe r
iii. The pressure gradient P/L causing the flow

$$\frac{V}{t} \propto \eta^x r^y \left(\frac{P}{l}\right)^z$$

$$\frac{V}{t} = K\eta^x r^y \left(\frac{P}{l}\right)^z \text{..........................x}$$

$$\frac{[V]}{[t]} = [K][\eta]^x [r]^y \left(\frac{[P]}{[l]}\right)^z$$

K is a dimensionless constant

$$L^3T^{-1} = (M L^{-1}T^{-1})^x \; L^y \; (M L^{-2}T^{-2})^z$$

$$L^3T^{-1} = M^{x+z} L^{y-x-2z} \; T^{-x-2z}$$

For M, $0 = x + z$.........................1

For L, $3 = y - x - 2z$........................2

For T, $-1 = -x - 2z$.......................3

From equation 1: $0 = x + z$

$x = -z$

Put into equation 3: $-1 = -(-z) - 2z$

$-1 = -z$

$z = 1$

$x = -1$

Put into equation 2

$3 = y - -1 - 2$

$3 = y - 1$

$y = 4$

$x = -1, y = 4, z = 1$

But from x

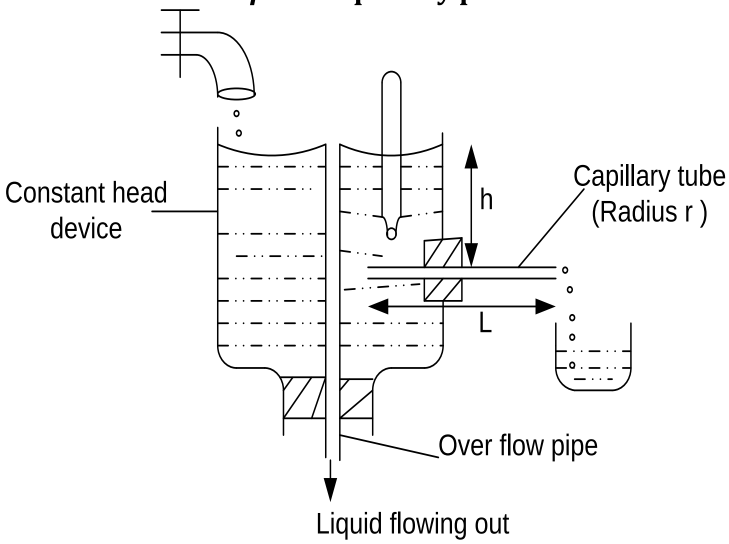
$$\frac{V}{t} = K\eta^x r^y \left(\frac{P}{l}\right)^z$$

$$\frac{V}{t} = \frac{K r^4 P}{\eta l}$$

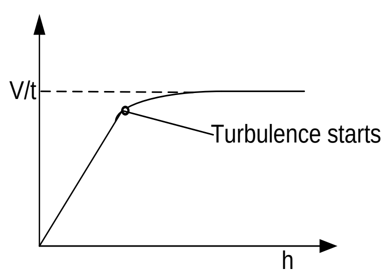
By experiment $K = \frac{\pi}{8}$

$$\boxed{\frac{V}{t} = \frac{\pi r^4 P}{8\eta l}}$$ - Poiseuille's formula

a: Measurement of η of a liquid by poiseuille's formula



- The liquid under test flows steadily through the capillary tube from a constant head device and the volume V of liquid which emerges in a known time t is collected in a beaked and measured.
- The radius r of the tube is measured using a travelling microscope and a meter rule is used to measure its length L and height h of the liquid above the tube.
- The procedure is repeated by lowering or raising the overflow pipe to obtain several values of $\left(\frac{V}{t}\right)$ for each h.
- A graph of $\left(\frac{V}{t}\right)$ against h is plotted and its slope is determined from the straight part of laminar flow.



$$From\ \frac{V}{t} = \frac{\pi r^4 P}{8 \eta l}$$

But P = hρg where $\rho$is the density of the liquid

$$\frac{V}{t} = \left(\frac{\pi r^4 \rho g}{8\eta l}\right) h$$

Comparing with y = mx +c

Slope S = $\left(\frac{\pi r^4 \rho g}{8\eta l}\right)$

$$\eta = \frac{\pi r^4 \rho g}{8\ l s}$$

Note:

- The experiment must be carried out at a constant temperature to avoid changes in η
- Constant head apparatus is used to ensure that the rate of liquid flowing through the capillary tube is uniform. Since Poisseulle's formular holds for only laminar flow

- Great care is needed when measuring r because it appears in the calculation of η as r^4. This makes the % error in η due to an error in r four times the % error in r
- A capillary tube is used because r needs to be small so that h is large enough to be measured accurately

11.2.0: STOKE'S LAW AND TERMINAL VELOCITY

11.2.1: Derivation of Stoke's law

Stoke's suggested that any particle moving through a fluid experiences a retarding force called **viscous drag** due to the viscosity of the fluid. This force depends on the speed of the body V and acts in opposite direction to its motion

Note:

Viscosity of a fluid is the frictional force opposing relative motion between adjacent layers while **viscous drag** is the frictional force experienced by a body moving in a fluid due to its viscosity.

The viscous drag F on a spherical body depends

- ✓ On the radius (r) of the sphere
- ✓ Velocity V of the sphere
- ✓ Coefficient of viscosity η

$F \propto \eta^x V^y r^z$

$F = K\eta^x V^y r^z$……….(x)

$[F] = [K][\eta]^x [V]^y [r^z]$

K is a dimensionless constant

$MLT^{-2} = (ML^{-1}T^{-1})^x (LT^{-1})^y (L)^z$

$MLT^{-2} = M^x L^{y-x+z} T^{-x-y}$

For M

$x = 1$………………..(1)

For L,

$1 = y - x + z$

$y + z = 2$………………(2)

For T,

$-2 = -x - y$

$-2 = -1 - y$

$y = 1$

Put into eqn 2

$y + z = 2$

$z = 1$

$x = 1, y = 1, z = 1$

From equation x

$F = K\eta^x V^y r^z$

F = k η Vr

Experiment showed that K = 6π

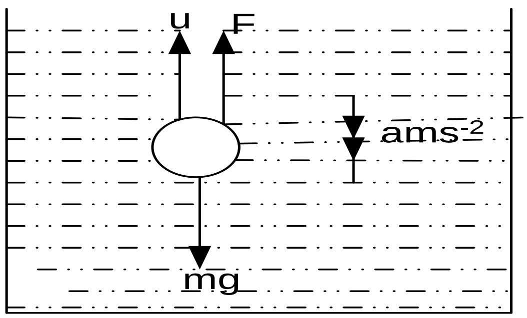
$\boxed{F = 6\pi\eta\ rV}$ **-Stoke's law**

11.2.2: TERMINAL VELOCITY

Consider a sphere of radius, r falling from rest through a viscous fluid.

- The forces acting on the sphere are its weight downwards , up thrust upwards due to the displaced fluid and the viscous drag, F upwards due to viscosity of the fluid.

- As the body accelerate downwards, its velocity increase and from $F = \pi \boldsymbol{\eta} r V_o$ so does the viscous drag increase .When the body is completely immersed in the fluid, up thrust remains constant since no more fluid is being displaced.
- A point is reached when $Mg = U + F$. This implies that the net force acting on the body is zero and body continues to move down with a maximum constant velocity called ***terminal velocity***.



If σ and ρ re the densities of the fluid and sphere respectively, the;

At the terminal velocity: $Mg = U + F$......(1)

$$\frac{4}{3}\pi r^3\rho g = \frac{4}{3}\pi r^3\sigma g + 6\pi \boldsymbol{\eta} r V_o$$

$$6\pi \boldsymbol{\eta} r V_o = \frac{4}{3}\pi r^3 g(\rho - \sigma)$$

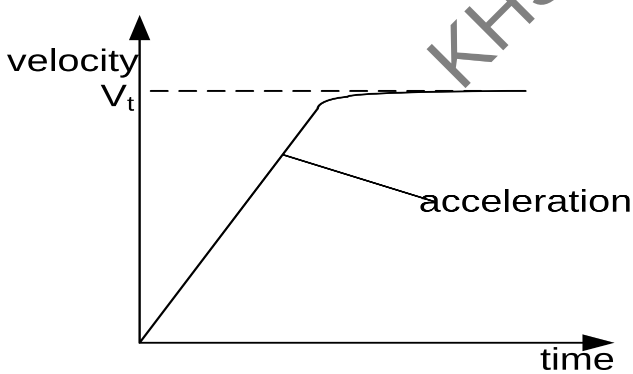
$$V_o = \frac{4\pi r^3 g(\rho-\sigma)}{3 x 6\pi\eta r}$$

$$V_o = \frac{2r^2 g(\rho-\sigma)}{9\eta}$$

Definition

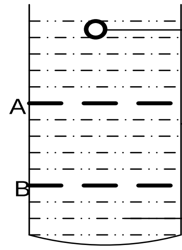
Terminal velocity is the maximum constant velocity attained by a body falling through aviscous fluid.

A graph of velocity against time for an object falling in a fluid



b: Measurement of η liquid by Stoke's law

The method is suitable for liquids of high viscosity such as glycerin and treacle



- Densities of the ball bearing and liquid ρ and σ respectively are obtained.
- Three reference marks A , B and C at equal distances are made on the sides of a tall transparent tube filled with the liquid.
- The ball is allowed to fall centrally through the liquid. The times t_1 and t_2 taken for the ball to fall from A to B and from B to C respectively are measured and noted .

When $t_1 = t_1 = t$, terminal velocity is obtained from

$V_o = \frac{AB}{t} = \frac{BC}{t} = \frac{AC}{2\,t}$…………………[1]

- The and diameter d and hence radius r of the ball bearing is measured using a micrometer screw gauge.

Coefficient of viscosity is then calculated from Stoke's using

$$V_o = \frac{2\,r^2\,g\,(\rho-\sigma)}{9\eta}$$

$\boldsymbol{\eta} = \frac{2\,r^2\,g\,(\rho-\sigma)}{9\,V_t}$………………………………[2]

Note:

i) A measuring cylinder which is wide compared with the diameter of the ball bearing.

ii) Point C should be far away from the top of the tube so that the temperature remains constant.

iii) using a highly viscous liquid and a small ball bearing makes t large enough to be measured

Question

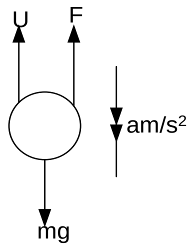
Describe how you can determine terminal velocity of a sphere falling in a viscous fluid.

An [The whole experiment of Stoke's law is the answer but only end in equation 1]

Numerical examples

1. A spherical raindrop of radius 2.0x10^{-4}m, falls vertically in air at 20^{0}c, if the densities of air and water are 1.3kgm^{-3} and 1x10^{3}kgm^{-3} respectively and the viscosity of air at 20^{0}c is 1.8x10^{-5}pa. Find the terminal velocity of the drop

Solution



At terminal velocity : Mg = U + F

$\frac{4}{3}\pi r^3\rho_s\, g = \frac{4}{3}\pi r^3\rho_f\, g + 6\pi\, \boldsymbol{\eta}\, rV_o$

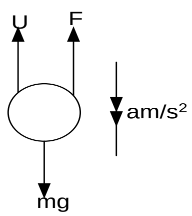
$V_o = \frac{2\, r^2\, g\,(\rho_f - \rho_s)}{9\,\eta}$

$V_o = \frac{2x\,(2x10^{-4})^2 x9.81x\,(1x10^3 - 1.2)}{9x1.8x10^{-5}}$

V_o= 4.84ms^{-1}

2. A spherical oil drop of density 900kgm^{-3} and radius 2.5x10^{-6}m has a charge of 1.6x10^{-19}C. the drop falls under gravity between two plates
    i. Calculate the terminal velocity attained by the drop
    ii. What electric field intensity must be applied between the plates in order to keep the drop stationary (density air = 1kgm^{-3}, coefficient of viscosity of air = 1.8x10^{-3}Nm^{-2}s^{-1})

Solution



At terminal velocity: Mg = U + F

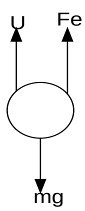
$\frac{4}{3}\pi r^3\rho_s\, g = \frac{4}{3}\pi r^3\rho_f\, g + 6\pi\boldsymbol{\eta}\, rV_o$

$V_o = \frac{2\, r^2\, g\,(\rho_f - \rho_s)}{9\eta}$

$V_o = \frac{2x\,(2.5x10^{-6})^2 x9.81x\,(900-1)}{9x1.85x10^{-5}}$

$V_o = 6.62x10^{-6} m/s$

Since the sphere is moving down, the electric field must be applied upwards to keep it stationary and there will be no vicious drag



When it is stationary Mg = U + F$_e$

$\frac{4}{3}\pi r^3\rho_s\, g = \frac{4}{3}\pi r^3\rho_f\, g + EQ$

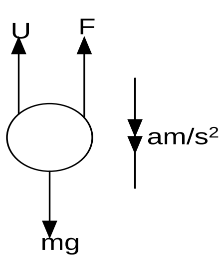
$E = \frac{4\pi r^3 g(\rho_f - \rho_s)}{3xQ}$

E = $\frac{4x\frac{22}{7}x(2.5x10^{-6})^3 x9.81x(900-1)}{3x1.6x10^{-19}}$

E = 3.60x10^{6}Vm^{-1}

3. Find the terminal velocity of an oil drop of radius 2.5x10^{-6}m which falls through air. Neglecting the density of air.(Viscosity of air = 1.8x10^{-5}Nm^{-2}, density of oil = 900kgm^{-3}**)**

Solution



At terminal velocity: Mg = U + F

$\frac{4}{3}\pi r^3\rho_s\, g = \frac{4}{3}\pi r^3\rho_f\, g + 6\pi\, \boldsymbol{\eta}\, rV_o$

$V_o = \frac{2\, r^2\, g\,(\rho_f - \rho_s)}{9\,\eta}$

But ρ_f= 0kgm^{-3}

$V_o = \frac{2x\,(2.5x10^{-6})^2 x9.81x\,(900-0)}{9x1.8x10^{-5}}$

$V_o = 6.81x10^{-4} m/s$

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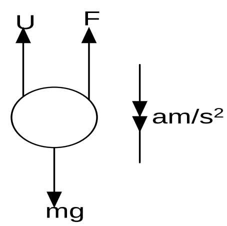
4. A metal ball of diameter 10mm is timed as it falls through oil at a steady speed, it takes 0.5s to fall through a vertical distance of 0.03m. Assuming that density of the metal is 7500kgm^{-3} and that of oil is 900kgm^{-3}, find
   i. The weight of the ball (2 marks)
   ii. The up thrust on the ball
   iii. The coefficient of viscosity of oil (03 marks

(Assume the viscous force = $6\pi \eta r V_0$ where η is the coefficient of viscosity, r is radius of *the ball and V_0 is terminal velocity)*

Solution



i) Weight = mg

$= \frac{4}{3}\pi r^3 \rho_s g$

$= \frac{4}{3} x \frac{22}{7} x (10x10^{-3})^3 x7500x9.81$

Weight = 0.31N

ii) Up thrust $U = \frac{4}{3}\pi r^3 \rho_f g$

$= \frac{4}{3} x \frac{22}{7} x (10x10^{-3})^3 x900x9.81$

U = 0.037N

iii) At terminal velocity Mg = U +F

$0.31 = 0.037 + 6\pi \eta r V_0$

$\eta = \frac{0.31-0.037}{6\pi r V_o}$

but $V_o = \frac{0.3}{0.5}$

$V_o = 0.6 m/s$

$\eta = \frac{0.31-0.037}{6x\frac{22}{7}x10x10^{-3}x0.6}$

η = 2.414Nsm^{-2}

Exercise:27

1. A small oil drop falls with terminal velocity of 4x10^{-4}ms^{-1} through air. Calculate the radius of the drop. What is the terminal velocity of oil drop if its radius is halved.
   (viscosity of air = 1.8x10^{-5}Nm^{-2}s, density of oil = 900kgm^{-3}, neglect density of air)
   An [1.92x10^{-6}m, 1.0x10^{-4}ms^{-1}]

2. A spherical rain drop of radius $2.0x10^{-4}$m falls vertically in air at 20°C. If the densities of air and water are $1.2kgm^{-3}$ and $1000kgm^{-3}$ respectively and that the coefficient of viscosity of air at 20°C is $1.8x10^{-5}Pa$ s, calculate the terminal velocity of the drop. **An[4.484 ms^{-1}].**

3. A metal sphere of radius 2.0x10^{-3}m and mass 3.0x10^{-4}kg falls under gravity, central down a wide tube filled with a liquid at 35^{0}c, the density of the liquid is 700kgm^{-3}, the sphere attains a terminal velocity of magnitude 40x10^{-2}ms^{-1}. The tube is emptied and filled with another liquid at the same temperature and of density 900kgm^{-3}. When the metal sphere falls centrally down the tube, it is

found to attain a terminal velocity of magnitude $25x10^{-2}ms^{-1}$. Determine at 35^0C, the ratio of the coefficient of viscosity of the second liquid to that of the first. **[an 1.640]**

4. In an experiment to determine the coefficient of viscosity of motor oil, the following measurements were made

Mass of glass of sphere = $1.2x10^{-4}kg$

Diameter of sphere = $4.0x10^{-3}$,

Terminal velocity of sphere = $5.4x10^{-2}ms^{-1}$

Density of oil = $860kgm^{-3}$

Calculate the coefficient of viscosity of the oil [**an 0.45Nsm^{-2}]**

5. A metal sphere of radius $3.0x10^{-3}m$ and mass $4.0x10^{-4}kg$ falls under gravity, central down a wide tube filled with a liquid at 25°C, the density of the liquid is $800kgm^{-3}$, the sphere attains a terminal velocity of magnitude $45cms^{-1}$. The tube is emptied and filled with another liquid at the same temperature and of density $100kgm^{-3}$. When the metal sphere falls centrally down the tube, it is found to attain a terminal velocity of magnitude $20cms^{-1}$. Determine at 25^0C, the ratio of the coefficient of viscosity of the second liquid to that of the first. **[An 2.09]**

6. A steel sphere of diameter $3.0x10^{-3}m$ falls through a cylinder containing a liquid x .When the sphere has attained a terminal velocity, it takes 1.08 s to travel between two fixed marks on the cylinder. When the experiment is repeated using another steel sphere of diameter $5.0x10^{-3}m$ with the cylinder containing liquid y, the time of fall between two fixed points is 4.8 s. if the density of liquid x is $1.26x10^3kgm^{-3}$, that of liquid y is$0.92x10^3kgm^{-3}$ and that of the steel ball is $7.8x10^3kgm^{-3}$, determine the ratio of the coefficient of viscosity of the liquid x to that of the liquid y, if the temperature remains constant throughout. **[An 0.77]**

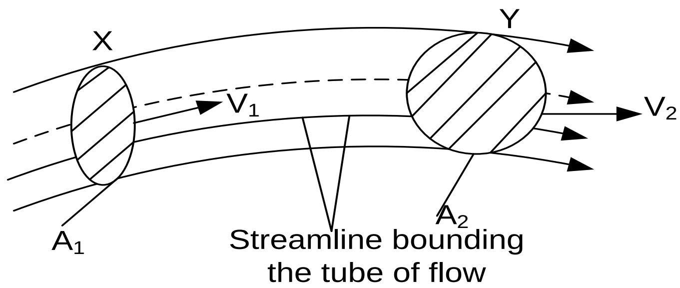
11.3.0: Equation of continuity

Consider a fluid undergoing steady flow and consider a section XY of a tube of flow with the fluid.

Let A_1 and A_2 be the cross-section areas of the tube of flow at X and Y respectively

$\rho_1$and ρ_2 be the densities of the fluid at X and Y respectively

V_1 and V_2 be the velocities of the fluid particles at X and Y respectively.



In a time interval Δt the fluid at X will move forward a distance $V_1\Delta t$. therefore, a volume $A_1V_1\Delta t$ will enter the tube at X. the mass of fluid entering at X in time Δt will be there be

$$\rho_1 A_1V_1\Delta t$$

Similarly the mass leaving at Y in the same time is $\rho_2 A_2V_2\Delta t$

Since the mass entering at X is equal to mass leaving at Y

$$\rho_1A_1V_1\Delta t = \rho_2 A_2V_2\Delta t$$

For an incompressible fluid $\rho_1 = \rho_2$

$$\boxed{\boldsymbol{A_1V_1 = A_2V_2}} \text{.......................................1}$$

Equation 1 is an equation of continuity for an incompressible fluid

Definition

An incompressible fluid is a fluid in which changes in pressure produce no change in the density of the fluid

11.3.1: WHY LIQUIDS FLOW FASTER IN CONSTRICTIONS

Volume flow per second is constant, so by the equation of continuity: $\mathbf{A_1V_1 = A_2V_2}$

$V_2 = \frac{A_1}{A_2}V_1$ It implies that $A_2 \propto \frac{1}{V_2}$ if $A_1 > A_2$ then $V_2 > V_1$

Hence the velocity at the wider part is less than that at the constructed part

11.3.2: Bernoulli's equation

It states that for an incompressible non viscous fluid undergoing steady flow, the pressure plus the potential energy per unit volume plus kinetic energy per unit volume is constant at all points on a stream line.

i.e. $\boxed{\boldsymbol{P + \tfrac{1}{2}\rho v^2 + \rho g h = a\ constant}}$

P is the pressure with in the fluid

ρ is the density of the fluid

v is the velocity of the fluid

g is the acceleration due to gravity

h is height of the fluid (above reference line)

11.3.3: Derivation of Bernoulli's equation

Consider a tube of flow with in a non-viscous incompressible fluid undergoing steady flow

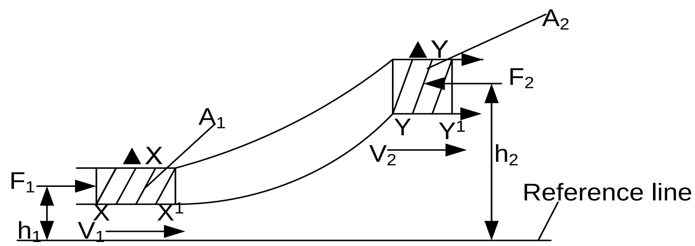
Let

ρ_1 and ρ_2 = pressure at X and Y

V_1 and V_2 = velocities at X and Y

A_1 and A_2 = area of cross section at X and Y

h_1 and h_2 = Average heights at X and Y



Let X^1 be close to X so that each of the parameters above has the same value at X^1 and at X. Let Y^1 be close to Y with similar consequences

Since the fluid is incompressible, the density will be the same at all points. Let this be ρ.

Consider the section of fluid which is between X and Y moving to occupy the region between X^1 and Y^1. The fluid moves in this direction because the force F_1 is greater than the force F_2. The force F moves a distance Δx and the fluid moves a distance Δy against the force F_2

Network done on the fluid is therefore given by

$\boldsymbol{W = F_1 \Delta x - F_2 \Delta y}$.......................................1

Since the fluid is undergoing ste4ady flow, the mass of fluid that was originally between X and X^1 is equal to the mass which is now between Y and Y^1. Let this mass be M, thus a mass M which originally had velocityV_1 and average height H_1 has been replaced by an equal mass with velocity V_2 and average height h_2, therefore,

Gain in kinetic energy $= \frac{1}{2} m\, {v_2}^2 - \frac{1}{2} m\, {v_1}^2$.....................2

Gain in potential energy $= mgh_2 - mgh_1$**........................**3

Name of the work done on the fluid has been used to overcome internal friction since the fluid is non-viscous and therefore by the principle of conservation of energy.

Work done= Gain in kinetic energy + Gain in potential energy

Work done= $\frac{1}{2} m\, {v_2}^2 - \frac{1}{2} m\, {v_1}^2 + mgh_2 - mgh_1$

$F_1\, \Delta x$ - $F_2\, \Delta y = \frac{1}{2} m\, {v_2}^2 - \frac{1}{2} m\, {v_1}^2 + mgh_2 - mgh_1$

$P_1 A_1 V_1 \Delta t$ - $P_2\ A_2 V_2 \Delta t = \frac{1}{2} m\, {v_2}^2 - \frac{1}{2} m\, {v_1}^2 + mgh_2 - mgh_1$

But $A_1\, \Delta x$ (volume between X and X_1) **=** $\boldsymbol{\frac{m}{\rho}}$

And similarly volume between Y and Y^1 ($A_2\, \Delta y$) **=** $\boldsymbol{\frac{m}{\rho}}$

$$\rho_1 \frac{m}{\rho} - \rho_2 \frac{m}{\rho} = \frac{1}{2} m\, v_2{}^2 - \frac{1}{2} m\, v_1{}^2 + mgh_2 - mgh_1$$

$$P_1 - P_2 = \frac{1}{2} \rho\, v_2{}^2 - \frac{1}{2} \rho\, v_1{}^2 + \rho g h_2 - \rho g h_1$$

$$P_2 + \frac{1}{2} \rho\, v_2{}^2 + \rho g h_2 = P_1 + \frac{1}{2} \rho\, v_1{}^2 + \rho g h_1$$

$P + ½ \rho v^2 + \rho g h$ =aconstant

Assumptions

- ✓ The flow is laminar
- ✓ The fluid is incompressible and non viscous
- ✓ The pressure and velocity are uniform at any cross section of the tube

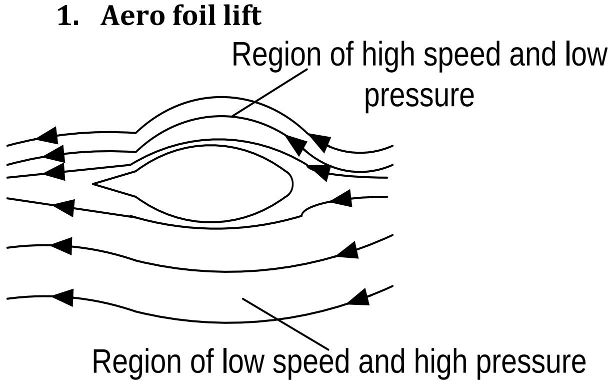
Note

In accordance with equation of continuity, fluids speed up at constrictions and therefore there is a decrease in pressure at constrictions. This effect is made use of in such devices are filter pumps, Bunsen burners and carburetors

11.3.4: Application of Bernoulli's principle

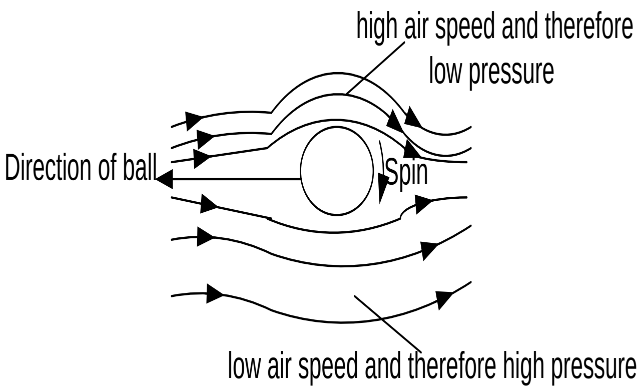
It follows from Bernoulli's equation that whenever a flowing fluid speeds up, there is a corresponding decrease in the pressure and for the potential energy of the fluids. If the flow is horizontal, the whole of the velocity increase is accounted for by a decrease in pressure.

1. **Aero foil lift**



- ❖ An aero foil e.g. an air craft wing is shaped so that air flows faster along the top of the wings than below the wings.
- ❖ By Bernoulli's principle pressure below becomes greater than that above the wings.
- ❖ This pressure difference produces the resultant force called lift upwards force. It is this force which provides a force that lifts the plane off the ground at take off

2. **A spinning ball**

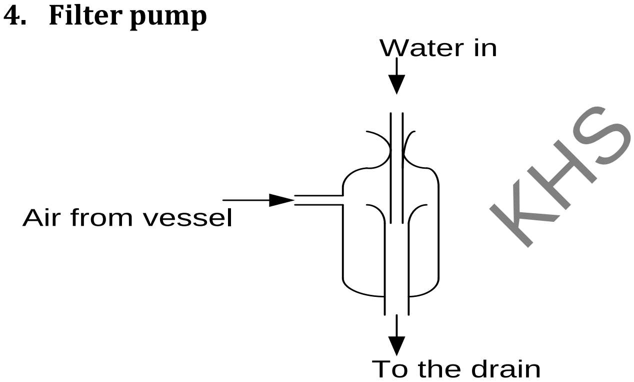


A ball such as a football, tennis or golf ball that is projected to travel through air experiences a sideways force which makes it curve in flight. This is because the spin drags air around with the ball such that air moves faster on one side of the ball than the other. The pressure difference causes a resultant force which makes the ball the curve as it spins.

3. **Sanction effect**

This is experienced by a person standing close to the platform at the station when a fast moving train passes. The fast moving air between the person and the train produces a decrease in pressure and the excess pressure on the other side pushes the person towards the train

4. **Filter pump**



This pump has a narrow cross section in the middle so that the jet of water from the top flows faster here. This reduces the pressure around it and thus air flows in from the side of a tube connected to a vessel. The air and water are expelled together through the bottom of the pump

5. **Bunsen burner**

The gas passes the narrow jet at high speed creating a low pressure region. Atmospheric pressure then pushes air in through the hole and the mixture flows up the tube to burn at the top

6. **Carburetor**

The air passage through a carburetor is partially constructed at the point where petrol and air are mixed. This increases the speed of air but lowers its pressure and permits more rapid evaporation of the petrol.

Examples

1. Water flows along a horizontal pipe of cross section area 30cm^2. The speed of water is 4ms^{-1} but this rises to 7.5m/s in constriction pipe. What is the area of this narrow part of the tube.

Solution

From the equation of continuity

$A_1V_1 = A_2V_2$

$30x10^{-4}x4 = A_2x7.5$

$A_2 = 1.6x10^{-3}m^2$

Area of the narrow part is $16cm^2$

2. Water leaves the jet of a horizontal horse at 10m/s. If the velocity of the water with in the horse is 0.4m/s. Calculate the pressure P with in the horse (density of water $1000kgm^{-3}$) and atmospheric pressure 10^5Nm^{-2}

Solution

$V_1 = 0.4m/s$, P_1 = ?, $1000kg/m^3$,

$V_2 = 10m/s$, $P = 10^5$

$P + ½ \rho v^2 + \rho gh$ = a constant

$P_1 + ½ \rho v_1^2 = P_2 + ½ \rho v_2^2$ for horizontal flow

$P_1 + ½ x1000x0.4^2 = 10^5 + ½ x1000x10^2$

$P_1 = 1.5x10^5Pa$

3. A fluid of density $1000kgm^{-3}$ flows in a horizontal tube. If the pressure the entry of the tube is 10^5Pa and at the exit is 10^3Pa, given that the velocity of the fluid at the entry is $8ms^{-1}$, calculate the velocity of the liquid at the exit.

Solution

$P + ½ \rho v^2 + \rho gh$ = a constant

$P_1 + ½ \rho v_1^2 = P_2 + ½ \rho v_2^2$ for horizontal flow

$10^5 + ½ x1000x8^8 = 10^3 + ½ x1000xV_2^2$

$V_2 = 45.2ms^{-1}$

4. An air craft design requires a dynamic lift of $2.4x10^4N$ on each square meter of the wing when the speed of the air craft through the air is $80ms^{-1}$. Assuming that the air flows past the wing with streamline line flow and that the flow past the lower surface is equal to the speed of the air craft, what is required speed of the air over the upper surface of the wing if the density of the air is $1.29kgm^{-3}$.

Solution

$P_1 + ½ \rho v_1^2 = P_2 + ½ \rho v_2^2$ for horizontal flow

$P_2 - P_1 = \frac{1}{2}\rho(V_1^2 - V_2^2)$

$P_2 - P_1 = \frac{1}{2}x1.29x(V_1^2 - 80^2)$

lift force, $F = (P_2 - P_1)A$

$24000 = \left[\frac{1}{2}x1.29x(V_1^2 - 80^2)\right]x1$

$V_1 = 208.8ms^{-1}$

5. Air flows over the upper surface of the wings of an aero plane at a speed of $81ms^{-1}$ and past the lower surfaces of the wings at $57ms^{-1}$. Calculate the lift force on the aero plane if it has a total wing area of $3.2m^2$. (density of air = $1.3kgm^{-3}$)

Solution

$P_1 + ½ \rho v_1^2 = P_2 + ½ \rho v_2^2$ for horizontal flow

$P_2 - P_1 = \frac{1}{2}\rho(V_1^2 - V_2^2)$

$P_2 - P_1 = \frac{1}{2}x1.3x(81^2 - 57^2)$

lift force, $F = (P_2 - P_1)A$

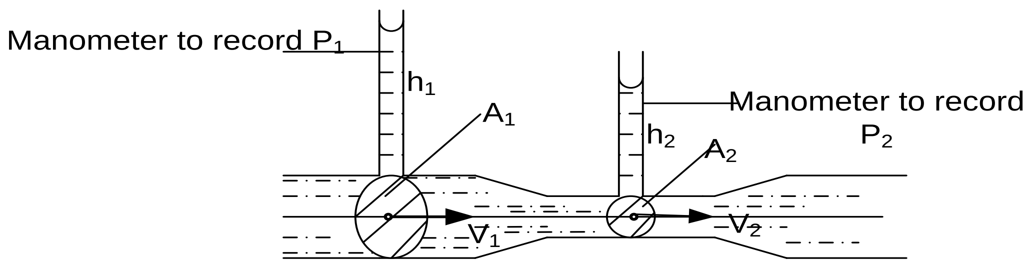
$$F = \left[\frac{1}{2}x1.3x(81^2 - 57^2)\right]x3.2$$

$$F = 6.9x10^3 N$$

11.3.5: Measurement of fluid velocity

5. Venture meter

This is a device which introduces a constriction into a pipe carrying a fluid in order that the velocity of the fluid can be measured by measuring the resulting drop in pressure.



Consider the fluid to be non viscous, incompressible and of density ρ in a horizontal steady flow let the pressure and velocity be P_1 and V_1 at the main pipe and P_2 and V_2 at the constricted pipe along the same stream line

Applying Bernoulli's equation

$\mathbf{P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2}$ ……………….(1) (horizontal flow)

If the cross sectional areas at main and constructed equation of continuity.

$A_1V_1 = A_2V_2$

$V_2 = \frac{A_1V_1}{A_2}$

Put into equation 1

$\mathbf{P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho \left(\frac{A_1V_1}{A_2}\right)^2}$

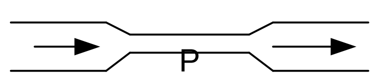
$$\mathbf{P_1 - P_2} = \frac{1}{2}\rho\, v_1{}^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

Thus by measuring pressures $\mathbf{P_1}$ and $\mathbf{P_2}$ and knowing ρ, A_1, and A_2 it is possible to find the velocity of V_1 of the fluid in the un constricted (main) section of the pipe.

Note: $P_1 = \rho h_1 g$ and $P_2 = \rho h_2 g$

Examples

1.a)



a horizontal pipe of a diameter 36.0cm tapers to a diameter of 18.0cm at P. An ideal gas at a pressure of $2x10^5$Pa is moving along the wider part of the pipe at a speed of $30ms^{-1}$, the pressure of the gas at P is $1.8x10^5$Pa. Assuming the temperature of the gas remain constant calculate the speed of the gas at P.

b) For the gas in (a) recalculate the speed at P on the assumption that it can be treated as an incompressible fluid, and use Bernoulli's equation to calculate corresponding value for the pressure at P. Assume that in the wider part of the pipe the gas speed is still $30.0ms^{-1}$, the pressure is still $2.00x10^5$ Pa and at this pressure the density of the gas is $2.60kgm^{-3}$.

Solution

a) $P_1=2x10^5$Pa $d_1=36x10^{-2}$m, $v_1=30ms^{-1}$

$P_2=1.8x10^5$Pa $d_2=18x10^{-2}$m $v_2=?$

An ideal gas at constant temperature obeys Boyle's law.

$P_1V_1{}^1 = P_2V_2{}^1$ ---------------------------- [1]

volume $V_1{}^1 = A_1L_1$ and volume $V_2{}^1 = A_2L_2$

But L_1 = speed V_1 x t and L_2= speed V_2 x t

Put into equation 1 : $P_1A_1L_1t = P_2A_2L_2t$

$P_1\frac{\pi d_1{}^2}{4}L_1t = P_2\frac{\pi d_2{}^2}{4}L_2t$

$2x10^5x\frac{\frac{22}{7}x(36x10^{-2})^2}{4}x30 = 2x10^5x\frac{\frac{22}{7}x(18x10^{-2})^2}{4}xV_2$

$V_2 = 133.33m/s$

b) For an incompressible fluid

$A_1V_1 = A_2V_2$---------------- [2]

$P_1=2x10^5$Pa $d_1=36x10^{-2}$m $v_1=30ms^{-1}$

$P_2=?$ $d_2=18x10^{-2}$m, $v_2=?$

$\frac{\pi d_1{}^2}{4}V_1 = \frac{\pi d_2{}^2}{4}V_2$

$\frac{22}{7}x\frac{(36x10^{-2})^2}{4}x30 = \frac{22}{7}x\frac{(18x10^{-2})^2}{4}xV_2$

$V_2 = 120m/s$

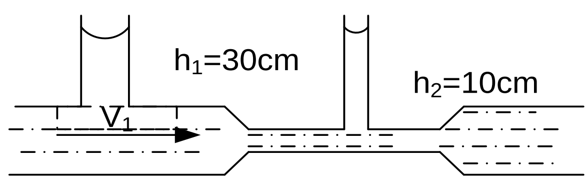
Using Bernoulli's equation

$P_1 + ½ \rho v_1{}^2 = P_2 + ½ \rho v_2{}^2$ for horizontal flow

$2x10^5 + ½ x2.6x30^2 = P_2 + ½ x2.6x120^2$

$P_2=1.825x10^5$Pa

6. A venturimeter consists of a horizontal tube with a constriction tube which replaces part of the piping system as shown below



If the cross-section area of the main pipe is $5.8x10^{-3}m^2$ and that of the constriction is $2.58x10^{-3}m^2$. Find the velocity V_1 of the liquid in the main pipe

Solution

$h_1 = 30x10^{-2}$, $h_2 = 10x10^{-2}$m, ρ_1 =? ρ_2 =?,

A_1 = 5.81x10-3m³, A_2 = 2.58x10-3m³

$P_1 = h_1\rho g$ and $P_2 = h_2\rho g$

P + ½ ρv^2 + ρgh = a constant

$P_1 + ½ \rho v_1{}^2 = P_2 + ½ \rho v_2{}^2$ for horizontal flow

$$\frac{1}{2}\rho v_1{}^2 + \rho gh_1 = \frac{1}{2}\rho v_2{}^2 + \rho gh_2$$

From equation of continuity

$A_1V_1 = A_2V_2 \quad \therefore V_2 = \frac{A_1V_1}{A_2}$

$\frac{1}{2}\rho\, v_1{}^2 + \rho g h_1 = \frac{1}{2}\rho\left(\frac{A_1V_1}{A_2}\right)^{2} + \rho g h_2$

$30x10^{-2}x9.81 + \frac{1}{2}xV_1^2 = 10x10^{-2}x9.81 + \frac{1}{2}x\left(\frac{5.81x10^{-3}xV_1}{2.58x10^{-3}}\right)^2$

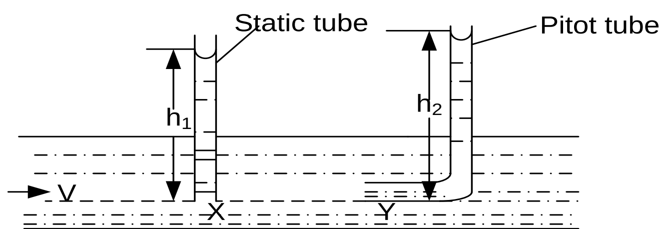
$2.943 - 0.981 = 2.035612343V_1^2$

$V_1^2 = 0.963837739$

$V_1 = 0.982m/s$

3. Pitot-static tubes

The Pitot - static tube is a device used to measure the velocity of a moving fluid. It consists of two manometer tubes, the pitot tube and the static tube. The pitot tube has its opening facing the fluid flow, the static tube has its opening at right angles to this



The total pressure exerted by a flowing liquid has two components ie the static pressure and dynamic pressure. Static tube measures the static pressure while pitot tube measures total pressure.

$P + ½\,\rho v^2 + \rho g h$ = a constant

Static pressure= $P + \rho g h_1$ and Dynamic pressure = $\frac{1}{2}\rho V^2$

$Total pressure = static pressure + dynamic pressure$

$Dynamic pressure = total pressure - static\ pressure$

$\frac{1}{2}\rho V^2 = total pressure - static\ pressure$

$V = \sqrt{\frac{2(total pressure - static\ pressure)}{\rho}}$

- **Static pressure**

Static pressure at a point is the pressure that the fluid would have if it were at rest.

- **Dynamic pressure**

It is the pressure of a fluid due to its velocity

- **Total pressure**

It is the sum of the dynamic and static pressure.

Example

1. The static pressure in a horizontal pipe line is $4.3 x 10^4$Pa, the total pressure is $4.7 x 10^4$Pa and the area of cross-section is 20cm^2. The fluid may be considered to be incompressible and non viscous and has a density of 10kgm^{-3}. Calculate
   i. The flow velocity in the pipeline
   ii. The volume flow rate in the pipeline

Solution

Static pressure =$4.3 x 10^4$Pa

Total pressure = $4.7 x 10^4$Pa

A = 20 x 10^{-4}m^2, P = 10kgm^{-3}

Dynamic pressure=total pressure–static pressure

Dynamic pressure =$4.7 x 10^4 - 4.3 x 10^4$

Dynamic pressure =0.4 x 10^4Pa

Dynamic pressure = $\frac{1}{2}\rho V^2$

0.4 x 10^4 = $\frac{1}{2} x 10^3 V^2$

V = 2.83m/s

ii) Volume flow rate = $\frac{volume}{time}$

$\frac{volume}{time} = \frac{A\,L}{time} = \frac{A\,v\,t}{t}$

= $20 x 10^{-4}$x 2.83

$\frac{volume}{time} = 5.66x10^{-3}m^3s^{-1}$

2.Water flows steadily along a uniform flow tube of cross-sectional area $30cm^2$. The static pressure is $1.20x10^5 Pa$ and the total pressure is $1.28x10^5 Pa$. assuming that the density of water is $1000kgm^{-3}$, calculate the;

(i) Flow velocity
(ii) Volume flux
(iii) Mass of water passing through a section of the tube per second

Solution

(i) $V = \sqrt{\frac{2(totalpressure - static\ pressure)}{\rho}}$

$V = \sqrt{\frac{2(1.28x10^5 - 1.20x10^5)}{1000}} = 4ms^{-1}$

(ii) volume per second = area x velocity

$= 30x10^{-4}x4$

Volume flux = $0.012m^3s^{-1}$

(iii) Mass per second= volume per second x ρ

$= 0.012x1000$

Mass per second= $12kgs^{-1}$

3. A pitot – static tube fitted with a pressure gauge is used to measure the speed of a boat at sea. Given that the speed of the boat does not exceed 10m/s and the density of sea water is 1050kgm^{-3}, calculate the maximum pressure on the gauge

Solution

Maximum pressure is the dynamic pressure

Dynamic pressure = $\frac{1}{2}\rho V^2$

= $\frac{1}{2}x1050x10^2$

Dynamic pressure =5.25x10^4Pa

Exercise: 28

6.Water flows speedily along a horizontal tube of cross-sectional area 25cm^2. The static pressure with in the pipe is 1.3x10^5pa and the total pressure 1.4x10^5pa. Calculate the velocity of the water flow and the mass of the water flow past a point in a tube per second. [**an 4.47m/s, 11.175kg/s]**

7. A lawn sprinkler has 20 holes each of cross sectional area 2x10^{-2}cm^2 and its connected to a horse pipe of cross sectional area 2.4cm^2, if the speed of the water in the horse pipe is 1.5m/s, estimate the speed of the water as it emerges from the holes. [**an9m/s]**

8. Water flows speedily along a uniform flow tube of cross section 30cm^2. The static pressure is 1.2x10^5Pa and the total pressure is 1.28x10^5Pa. Calculate the flow velocity and the mass of water per second flowing past a section of the tube. (Density of water is 1000kgm^{-3}**.)[an 4m/s, 12kg/s]**

9. Air flows over the upper surface of the wings of an aero plane at a speed of 120ms^{-1} and past the lower surfaces of the wings at 110ms^{-1}. Calculate the lift force on the aero plane if it has a total wing area of 20m^2. (density of air = $1.29kgm^{-3}$) **[an= 2.97x10^4N]**

11.4.0: FLUIDS AT REST

11.4.1: DENSITY AND RELATIVE DENSITY

Density of a substance is defined as the mass per unit volume of a substance.

$$\rho = \frac{m}{v}$$

S.I unit's kgm^{-3}

Relative density

Definition

It is the ratio of the density of a substance to density of an equal volume of water at 4^0c

It is at 4°C because water has maximum density of 1000kgm^{-3} at that temperature

$$R.D = \frac{density\ of\ asubstance}{density\ of\ water\ at\ 4°C}$$

$$R.D = \frac{m_s/v_s}{m_w/v_s}$$

$$R.D = \frac{m_s}{m_w}$$

It can also be defined as the ratio of the mass of a substance to mass of an equal volume of water

$R.D = \frac{m_s}{m_w}$ for W = mg $\frac{w_s/g}{w_w/g}$ R.D = $\frac{w_s}{w_w}$

It can also be defined as the ratio of weight of a substance to weight of an equal volume of water.

Note: Relative density has no units.

11.4.2: ARCHIMEDE'S PRINCIPLE

It states that when a body is wholly or partially immersed in a fluid, it experiences an up thrust equals to the weight of the fluid displaced.

I.e. Up thrust = weight of fluid = apparent loss of weight of the object in a fluid.

Definition

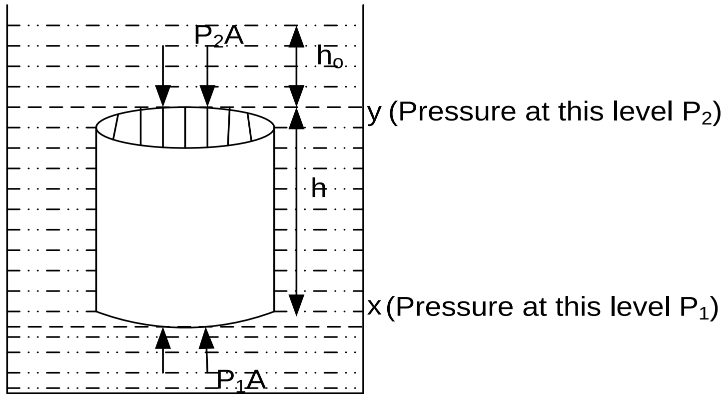
Up thrust is the apparent loss of weight of an object immersed in a fluid

Or

It is the resultant upward force on the body due to the fluid.

11.4.3: Verification of Archimedes' principle using a cylindrical rod

Consider a cylindrical rod of cross-sectional area A and height h immersed in a large quantity of a fluid of density ρ_f such that its top is at level Y, h_o meters below the surface of the fluid while its bottom is at level X shown below



Volume of fluid displaced = volume of cylinder

= Ah

Mass of fluid displaced = Ahρ_f

Weight of fluid displaced = Ahρ_f g…………………………..(i)

The fluid exerts forces of P_1A and P_2A on the bottom and top faces of the cylinder.

The up thrust (resultant upward force due to the fluid is therefore given by

$Upthrust$ = P_2A−P_1A

$Upthrust = (h + h_o)\ \rho_f\ gA - h_o\rho_f\ gA$

$Up\ thrust$= Ahρ_f g…………………………………………..(ii)

From equation (i) and equation (ii), therefore;

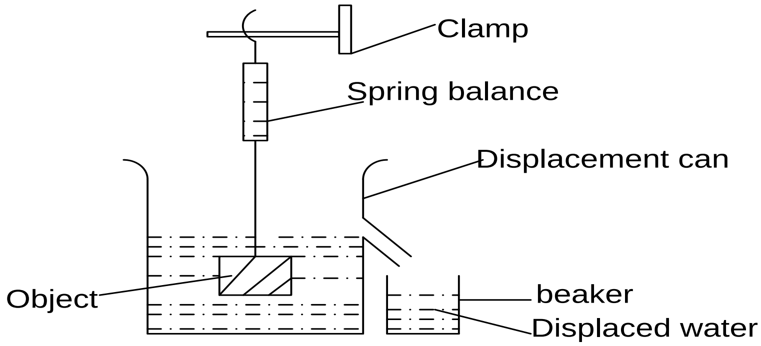
$Upthrust$ = weight of fluid displaced

Question: Show that the weight of fluid displaced by an object is equal to up thrust on the object

11.4.4: Verification of Archimedes' principle using a spring balance.

- Fill the displacement can with water till water flows through the spout and wait until the water stops dripping.
- Weigh a solid object in air using a spring balance and record its weight W_a
- Place a beaker of known weight beneath the spout of the can.

- With the help of the spring balance, the solid object is carefully lowered into the water in the displacement can and wait until water stops dripping when it is completely immersed, its weight (apparent weight) is then read and recorded from the spring balance as W_w.
- Re weigh the beaker and the displaced water and record the weight as $W_{(b+w)}$



Results

Let the weight of the empty beaker be W_b

Weight of water displaced = weight of (beaker +water) – weight of beaker

Weight of water displaced = $W_{(b+w)} - W_b$ ………………………….1

Apparent loss of weight of object = weight of object in air – weight of object in water

Apparent loss of weight of the object = W_a - W_w

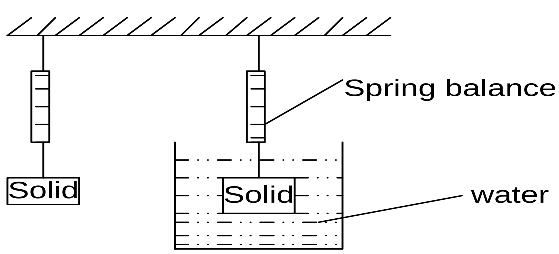
If (W_a - W_w) = ($W_{(b+w)} - W_b$), then Archimedes's principle is verified

11.4.5: Application of Archimedes' principle

It can be used to determine density and relative density of a solid and a liquid.

a) **Determination of density and relative density of a solid**

- Weigh the solid in air and its weight (W_a) recorded using a spring balance.
- Immerse the solid wholly in water and record its apparent weight from balance (W_w)



- Up thrust in water = W_a - W_w
- $R.D = \frac{Weight\ of\ a\ substance}{Up\ thrust\ in\ water}$
- $R.D = \frac{W_a}{W_a - W_w}$
- Density of solid = RD of solid x density of water

Example

1. An object suspended from the spring balance is found to have a weight of 4.92N in air and 3.87N when immersed in water. Calculate the density of the material from which the object is made of the density of water is 1000kgm^{-3}

Solution

W_a = 4.92, W_w = 3.87N

$$R.D = \frac{W_a}{W_a - W_w}$$

$$R.D = \frac{4.92}{4.92 - 3.87}$$

RD = 4.686

Density of substance = RD x ρ of water

= 4.686 x 1000

= 4686kgm^{-3}

Exercise : 29

1. A piece of glass weighs 0.5N in air and 0.30N in water. Find the density of the glass.

   An[2500kgm^{-3}]

2. A spherical stone has a mass of 1.546kg, if its radius is 20cm. find the density of the stone in

   (i) $g\ cm^{-3}$ (ii) $kg\ m^{-3}$

   An [46.848 $g\ cm^{-3}$, 4.6848 $kg\ m^{-3}$]

3. What is the mass of the sphere of diameter 20cm if its relative density is 14.1 **An[59.22kg]**

4. A glass block weighs 25N in air. When wholly immersed in water, the block weighs 15N. calculate

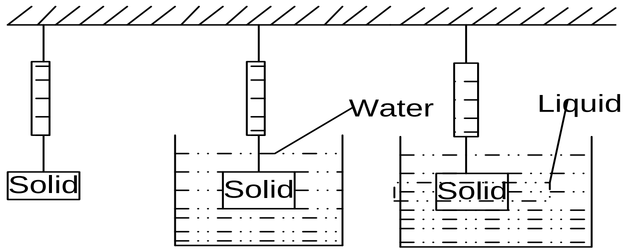
   i. The up thrust on the block
   ii. The density of the glass in $kg\ m^{-3}$ **An[10N, 2500 $kg\ m^{-3}$]**

b) Density of a floating object (eg cork)

The same experiment as above except the solid and the sinker are fixed together so that both become totally immersed in water.

c) Determination of density and relative density of a liquid

- Weigh a solid (sinker) in air and record its weight W_a using a spring balance.
- Immerse the solid (sinker) wholly in water and record the apparent weight W_w
- Wipe the surface of the solid (sinker) with a piece of dry cloth and immerse it wholly in the liquid whose relative density is to be measured, read and record its apparent weight in the liquid W_L



Weight of water displaced (up thrust in water) = W_a - W_w

Weight of liquid displaced (up thrust in liquid) = W_a - W_L

Relative density = $\frac{upthrust\ in\ Liquid}{upthrus\ in\ water}$

- $R.D\ of\ the\ liquid\ = \frac{W_a - W_L}{W_a - W_w}$

Density of liquid = R.D of liquid x density of water

Example

1. A solid has a weight of 160N in air and 120N when wholly immersed in a liquid of relative density 0.8, determine the density of a solid

Solution

R.D of Liquid = $\frac{Weight\ of\ solid\ in\ liquid}{Weight\ of\ equal\ volume\ of\ water}$

0.8 = $\frac{160-120}{Weight\ of\ solid\ in\ water}$

Weight of solid in water = $\frac{40}{0.8}$ = 50N

$R.D\ of\ solid = \frac{weight\ of\ solid\ in\ air}{weight\ of\ solid\ in\ water}$

$R.D\ of\ solid = \frac{160}{50}$

R.D of solid= 3.2

Density of a solid = RD of solid x $\rho$of water

= 3.2 x 1000

Density of a solid = 3200 kgm^{-3}

2. A piece of iron weighs 555N in air when completely immersed in water, it weighs 530N and weighs 535N when completely immersed in alcohol. Calculate the relative density of alcohol and the density of alcohol.

Solution

W_a = 555N W_w =530N W_L=535N

R.D of alcohol = $\frac{W_a - W_L}{W_a - W_w} = \frac{555-535}{555-530}$

R.D of alcohol =0.8

Density of alcohol = R.D of alcohol x ρ of H_2O

= 0.8 x1000

Density of alcohol = 800kgm^{-3}

3. A string supports a solid iron of mass 0.18kg totally immersed in a liquid of density 800 $kg\ m^{-3}$. Calculate the tension in the string if the density of iron is 800 $kg\ m^{-3}$

Solution

Weight of iron $= mg = 0.18x9.8 = 1.77N$

Volume of iron= $\frac{mas}{density} = \frac{0.18}{8000} = 2.25x10^{-5}m^3$

Mass of liquid displaced=2.25x10$^{-5}x8000$

$= 0.018kg$

Weight of the liquid displaced = $0.018x9.81$

Weight of the liquid displaced = 0.177N

At equilibrium ; mg = T+U

$1.77 = T + 0.1777$

$T = 1.593N$

4. A specimen of an alloy of silver and gold whose densities are 10.5gcm^{-3} and 18.9cm^{-3} respectively, weigh 3.2g in air and 33.13 g in water. Find the composition by mass of the alloy assuming that there has been no volume change in the process of producing he alloy. Assume that the density of water is 1 g cm^{-3}

Solution

$m_s + m_g = 325.2$……………………….....1

R.D of alloy = $\frac{35.2}{35.2-33.13}$ =17

Density of alloy= R.D x density of water

Density of alloy= 17x1=17 g cm^{-3}

Volume of alloy= $\frac{m}{\rho} = \frac{35.2}{17} = 2.07cm^3$

Volume of alloy= $V_s + V_g$

Volume of alloy= $\frac{m_s}{\rho_s} + \frac{m_g}{\rho_g}$

2.07= $\frac{m_s}{10.5} + \frac{m_g}{18.9}$…………………2

Solving 1 and 2 simultaneously

$m_s = 30.3g \ and \ m_g = 4.9g$

Exercise: 30

1. A block of mass 0.1kg is suspended from a spring balance when the block is immersed in water of density 1000kgm^{-3}, the spring balance reads 0.63N. When the block is immersed in a liquid of unknown density the spring balance reads 0.7N, find
   i) Density of the solid
   ii) Density of the liquid **An [2800kgm^{-3}, 800kgm^{-3}]**

2. An alloy contains two metals X and Y of densities $3.0x10^3kgm^{-3}$ and $5.0x10^3kgm^{-3}$ respectively. Calculate the density of the alloy if,
   (i) The volume of X is twice that of Y
   (ii) The mass of X is twice that of Y
   An[(i)=$3.7x10^3kgm^{-3}$(ii)= $3.5x10^3kgm^{-3}$]

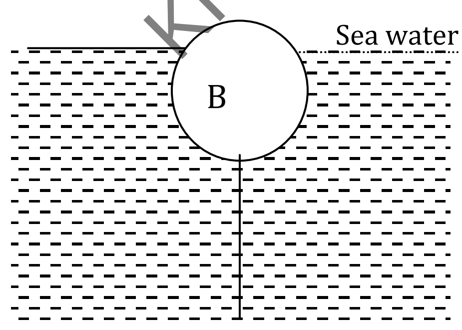
3. An alloy contains two metals A and B, has a volume of $5.0x10^{-4}m^3$ and a density of $5.6x10^3kgm^{-3}$. The densities of A and B are $8.0x10^3kgm^{-3}$ and $4.0x10^3kgm^{-3}$ respectively. Calculate the mass of A and mass of B. **An [A= 1.6kg, B=1.2kg]**

4. A piece of glass has a mass 62 kg in air. It has a mass of 32kg when completely immersed in water and a mass of 6kg when completely immersed in an acid.
   (a) The glass (b) The acid in $kg\ m^{-3}$

   An[(a)=1550 $kg\ m^{-3}$ (b)= 1400 $kg\ m^{-3}$]

5. A body of mass 0.1kg and relative density 2 is suspended by a thread and completely immersed in a liquid of density $920kgm^{-3}$.
   i) Find the tension in the thread. **An[0.53N]**
   ii) If the thread breaks, what will be the initial acceleration? **An [5.3ms^{-2}]**

6. A body of weight 0.52N in air weighs only 0.32N when totally immersed in water while its weight when immersed in another liquid is 0.36N. What is the density of the second liquid if the density of water is $1000kgm^{-3}$?
7. The figure below shows a buoy B, of volume 40 litres and mass 10kg. It is held in position in sea water of density $1.04g/cm^3$ by a light cable fixed to the bottom so that ¾ of the volume of the buoy is below the surface of the sea water.



   (i) Name the three forces that keep the buoy in equilibrium and state the direction in which each acts
   (ii) Determine the tension in the cable

8. A tank contains a liquid of density $1200kgm^{-3}$. A body of volume $5x10^{-3}m^3$ and density $900kgm^{-3}$ is totally immersed in the liquid and attached to by a thread to the bottom of the tank. Find the tension in the thread. **An [14.72N]**

9. A block of metal weighs 50N in air and 25N in water
   (a) Determine the density of the metal in $kg\ m^{-3}$
   (b) Find the weight of the metal in paraffin whose relative density is 0.8

   An[2000 $kg\ m^{-3}$, 30N]

11.5.0: FLOATATION

A body floats in a liquid if its density is less than the density of the liquid.

11.5.1: Law of floatation

It states that a floating body displaces its own weight in the fluid in which its floating.

Experiment to verify the law of floatation

(Same as verification of Archimedes' principle)

Note:

1. For a floating body
   - The weight of floating body =weight of fluid displaced
   - The weight of fluid =Up thrust
   - The weight of floating body = Up thrust
   - The mass of the floating body = the mass of the fluid displaced
   - A floating body sinks deeper in liquids of less density than in liquids of higher densities.
2. Density of a floating body = fraction submersed x density of liquid
3. Volume of displaced liquid = fraction submerged x volume of floating body.

Example

1. A solid weighs 237.5g in air and 12.5g when totally immersed in a fluid of density $0.9 g/cm^3$.
   Calculate
   a) Density of the solid.
   b) The density of the liquid in which the solid would float with 1/5 of its volume exposed above the liquid surface.

Solution

a) W_a =237.5g W_L =12.5g

Up thrust in liquid $= W_a - W_L = 237.5 - 12.5$

Up thrust in liquid (mass of liquid displaced)

=225g

Volume of liquid displaced $= \frac{m}{\rho} = \frac{225}{0.9}$

Volume of liquid displaced $=250 cm^3$

Volume of solid $=250 cm^3$

Density of solid $= \frac{Mass\ of\ solid}{volume\ of\ solid} = \frac{237.5}{250}$

$=0.95 g/cm^3$

b) If $\frac{1}{5}$ of its volume is exposed, then $\frac{4}{5}$ of its volume is submerged.

Volume of liquid = fraction x volume of the solid submerged

$= \frac{4}{5} x 25 = 200 cm^3$

Mass of solid =237.5

Density of liquid = $\frac{237.5}{200}$ **=**1.19g/cm^3

OR

ρ of floating body = fraction submerged x ρ liquid

0.95 = $\frac{4}{5}$ x density of liquid

Density of liquid= $\frac{0.95x5}{4}$=1.19gcm^{-3}

2. A solid of volume 10^{-4}m^3 floats in water of density 10^{3}kgm^{-3} with $\frac{3}{5}$ of its volume submerged
   i) Find the mass of the solid
   ii) If the solid floats in another liquid with $\frac{4}{5}$ of its volume submerged. What is the density of the liquid?

Solution

a) V=10^{-4} ρ_ω =1000kgm^{-3}

Volume submerged = $\frac{3}{5}$

Volume of water displaced = $\frac{3}{5}x$ volume of solid = $\frac{3}{5}x$ 10^{-4} $= 6x10^{-5}\ m^3$

mass of displaced water = volume of water displaced x density of water = 6x10^{-5}x 1000 = $6x10^{-2}\ kg$

By law of floatation, mass of water displaced is equals to the mass of the solid

∴ Mass of solid = $6x10^{-2}\ kg$

b) Fraction submerged = $\frac{4}{5}$

Density of solid = $\frac{mass\ of\ solid}{volume\ of\ solid} = \frac{6\ x\ 10^{-2}}{10^{-4}}$ = 600kgm^{-3}

Density of solid = fraction submerged x density of liquid

600 = $\frac{4}{5}x$ density of liquid

Density of liquid = 750kgm^{-3}

Exercise: 31

1. A Ball with a volume of 32cm^3 floats on water with exactly half of the ball below the surface. What is the mass of the ball (density of water =1.0x10^{3}kgm^{-3}) **An [1kg]**

2. An object floats in a liquid of density 1.2x10^{3}kgm^{-3} with one quarter of its volume above the liquid surface. What is the density of the object**.** **An[900kgm^{-3}]**

3. A solid weighs 237.g in air and 212.5g when totally immersed in a liquid of density0.9gcm^{-3}. Calculate the;

(i) Density of the solid

(ii) Density of a liquid in which the solid would float with $^1/_5$ of tis volume exposed above the liquid surface. **An[9500 kgm^{-3}. 1190 kgm^{-3}].**

4. Object with a volume of $1.0\text{x}10^{-5}\text{m}^3$ and density $4.0\text{x}10\text{kgm}^{-3}$ floats on water in a tank of cross sectional area $1.0\text{x}10^{-3}\text{m}^2$
   a) By how much does the water level drop when the object is removed
   b) Show that this decrease in water level reduces the force on the base of the tank by an equal amount to the weight of the (density of water = $1.0\text{x}10^3\text{kgm}^{-3}$) **An [$4\text{x}10^3\text{kgm}^{-3}$]**

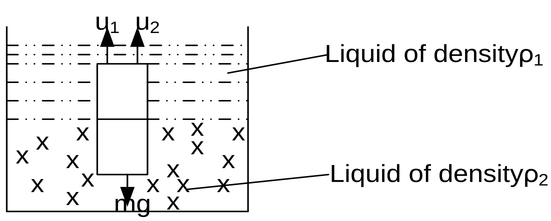
5. A block of wood floats in water of density 1000 kgm^{-3} with $^2/_3$ of its volume submerged . in oil it has $^9/_{10}$ of its volume submerged . find the densities of wood and oil

   An[740.74 kgm^{-3}, 666.67 kgm^{-3}].

6. A piece of metal of mass 2.60g and density 8.4 g cm^{-3} is attached to a block of wax of 1.0 g and density 0.92 g cm^{-3}. When the system is placed in a liquid, it floats with wax just submerged. Find the density of the liquid. **An [$1.13\text{x}10^{-6}$ g cm^{-3}]**

11.5.2: RELATION BETWEEN DENSITIES AND VOLUME FOR AN OBJECT FLOATING IN TWO LIQUIDS

Consider an object which floats at the interface between two immiscible liquids of density ρ_1 and ρ_2 with the objects having a density of $\boldsymbol{\rho}$



V_1 volume submerged to liquid of density ρ_1

V_2 e volume submerged to liquid of density ρ_2

U_1 the upthrust in liquid of density ρ_1

U_2 the upthrust in liquid of density ρ_2

Weight of object = Total upthrust

$$mg = U_1 + U_2$$

$$\rho(V_1 + V_2)g = \rho_1 V_1 g + \rho_2 V_2 g$$

$$\rho V_1 g + \rho V_1 g = \rho_1 V_1 g + \rho_2 V_2 g$$

$$\rho V_1 g - \rho_1 V_1 g = \rho_2 V_2 g - \rho V_2 g$$

$$\frac{V_1}{V_2} = \frac{\rho_2 - \rho}{\rho - \rho_1}$$

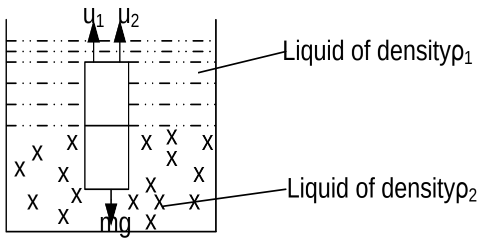
Note

When the object is replaced by a denser one, then the new object will sink much deeper and therefore V_1 decreases and V_2 increases by the same magnitude.

EXAMPLE: UNEB 2006 Q.4 (iii)

A block of wood floats at an interface between water and oil with 0.25 of its volume submerged in the oil. If the density of the wood is $7.3x10^2$kgm^{-3} . Find the density of the oil.

Solution



$V_1 = 0.75$ $V_2 = 0.25$

ρ =730kgm^{-3} ρ_1 =1000kgm^{-3} ρ_2 = ?

$$\frac{V_1}{V_2} = \frac{\rho_2 - \rho}{\rho - \rho_1}$$

$$\frac{V_1}{V_2} = \frac{\rho_2 - \rho}{\rho - \rho_1}$$

$$\frac{0.75}{0.25} = \frac{\rho_2 - 730}{730 - 1000}$$

$$3x(730 - 1000) = \rho_2 - 730$$

$$\rho_2 = -810 + 730$$

$\rho_2 = -80$kgm^{-3}

ρ_2 =80kgm^{-3}

Applications of law of floatation

1- Balloons 2- Ships 3- Submarines

- **Balloons**

A balloon filled with a light gas such as hydrogen gas rises up because the weight of the displaced air is greater than the weight of the balloon plus its content. It's the net upward force (up thrust) which pushes the balloon upwards and the balloon continues rising until the up thrust acting on it becomes equal to the weight of the balloon plus its content then it begins floating.

$\therefore$ U $= W_b + W_h + W_L$

$V_a \rho_a$g $= M_b$g+ $V_h\rho_h$g+ M_Lg

U=Up thrust	M_b=mass of balloon	ρ_a =density of air
W_h =weight of hydrogen	M_L=mass of load	ρ_h =density of hydrogen
W_b =weight of balloon	V_a =volume of air	
W_L =weight of load	V_h=volume of hydrogen	

Note

Volume of air displaced = volume of balloon

$$V_a = V_b$$

EXAMPLES

1. A balloon has a capacity of 10m^3 and is filled with hydrogen. The balloon's fabric and the container have a mass of 1.25kg. Calculate the maximum mass the balloon can lift .

   [ρ=0.089kgm^{-3}, ρ of air=1.29 kgm^{-3}]

Solution

V_b=10m³ ρ_h =0.089, ρ_a =1.29 kgm⁻³.

M_b=1.25 V_a=10m³ V_b=10m³

But up thrust = weight of balloon + weight of hydrogen +load

U= $W_b + W_h + W_L$

$V_a \rho_a g = M_b g + V_h \rho_h g + M_L g$

10x1.29g=1.25g+10x0.089+ M_Lg

M_L =10.76kg

2. A hot air balloon has a volume of 500m³. The balloon moves upwards at a constant speed in air of density 1.2kgm⁻³ when the density of the hot air inside it is 0.80kgm⁻³.
   a) What is the combined mass of the balloon and the air inside it.
   b) What is the upward acceleration of the balloon when the temperature of the air inside it has been increased so that its density is 0.7kgm⁻³.

Solution

V_b=500m³ V_h=500m³ V_a=500m³

ρ_a =1.2kgm⁻³ ρ_h =0.8kgm⁻³

U=W_b+W_h+W_L

$V_a \rho_a g = M_b g + V_h \rho_h g + M_L g$

$V_a \rho_a g = (M_b + M_L)g + V_h \rho_h g$

500x1.2g=(M_b+M_L)g +$M_h$xg

500x1.2=$(M_b+M_L+M_h)$

600 =$(M_b+M_L+M_h)$

Combined mass = 600kg

b) V_b=500m³ V_h=500m³ V_a=500m³

ρ_a =1.2kgm⁻³, ρ_h =0.8kgm⁻³

At equilibrium : U=W_b+W_h+W_L

$V_a \rho_a g = M_b g + V_h \rho_h g + M_L g$

500x1.2x9.81= (M_b+M_L)x9.81+500x0.8x9.81

(M_b+M_L)=200kg

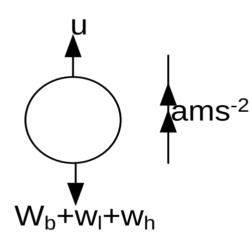
when ρ_h =0.7kg/m³, V_h=500

$W_h = V_h \rho_h g = 500x0.7x9.81 = 3433.5N$

$(W_b+W_L) = (M_b+M_L)x9.81$

$W_b+W_L = 200x9.81$ =1962N

U= $V_a \rho_a g = 500x1.2x9.81 = 5886$N



U − (W_b+W_h+W_L)= ma

$5886 - (1962 + 3433.5) = 600xa$

$a = 0.82ms^{-2}$

11.6.0: PRESSURE

The pressure acting on a surface is defined as the force per unit area acting normally on the surface

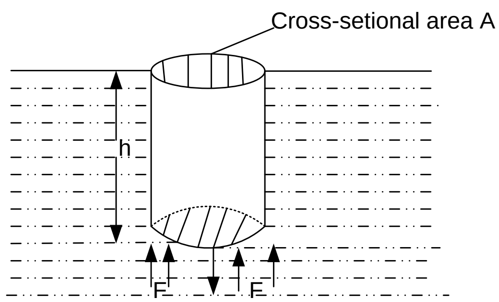
$$P = \frac{F}{A}$$

PRESSURE IN FLUIDS

The pressure in a fluid increased with depth, and all points at the same depth in the fluid are at the same pressure.

11.6.1: RELATION OF PRESSURE P WITH DEPTH h

Consider a cylindrical region of cross sectional area A and height h in a fluid of density ρ



The top of the cylinder is at the surface of the fluid and the vertical forces acting on it are its weight (mg) and an upward force F due to press p at the bottom of the cylinder.

The cylinder is in equilibrium and therefore

$F = mg$-------------------------[1]

But: $m = v\rho$ and $v = Ah$

F= Ahρg---------------------------- [4]

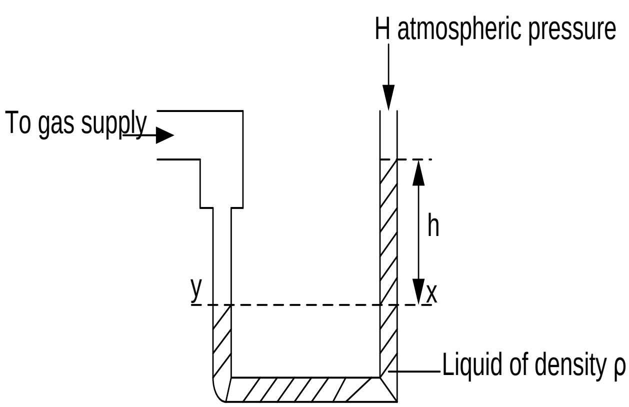
But P= $\frac{F}{A} = \frac{A\,h\,\rho\,g}{A}$

$P = h\rho g$

11.6.2: PRESSURE OF A GAS [U-TUBE MANOMETER]

This consists of a U-shaped tube containing a liquid. It is used to measure pressure.

The pressure to be measured (i.e. that of a gas) is applied to one arm of the manometer and the other arm is open to the atmosphere.



The gas pressure p is the same at the pressure at y

But pressure at y = pressure at x

$P = H + h\rho g$

Where $H = 1.01x10^5 Pa$

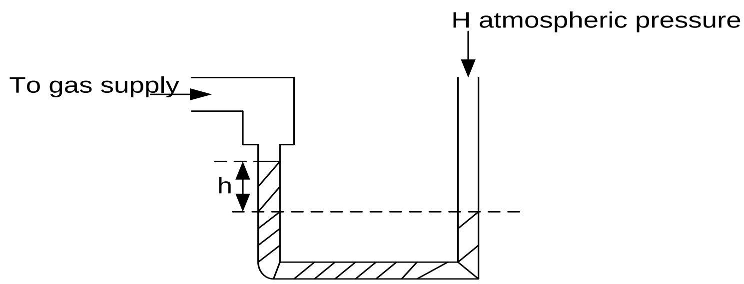
Or H=760mmHg

Or H=76cmHg

Note

The pressure recorded by the manometer (hρg) is known as gauge pressure. The actual pressure $(H + h\rho g)$ is called absolute pressure.

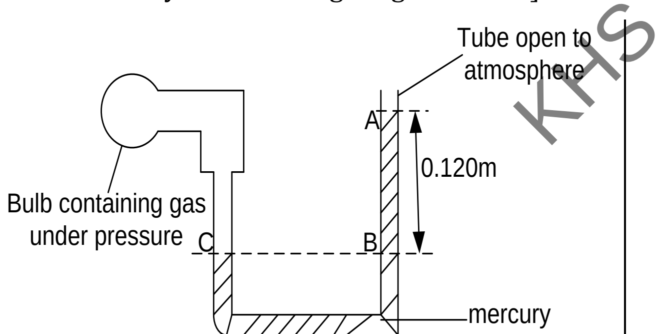
Suppose the level of the liquid in open limb of the manometer is lower than the level of the other side connected to a gas.



Pressure of gas $P = H - h\rho g$

Examples

1. Calculate the pressure of the gas in the bulb [Atmospheric pressure = 1.01x10^5Pa] density of mercury =1.30x10^4kgm^{-3} g=9.81ms^{-2}] Given the figure below;



Solution

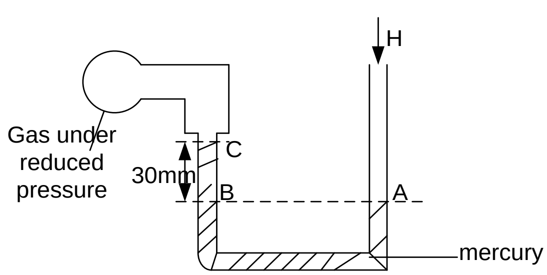
Pressure at C = pressure at B

Pressure at C =$H + h\rho g$

=1.01x10^5+$(0.12x1.36x10^4x9.81)$

Pressure of gas = 1.17x10^5Pa

2. Using the diagram below, calculate the pressure of the gas in the bulb. (atmospheric pressure =760mmHg)



Pressure at B = pressure at A =760mmHg

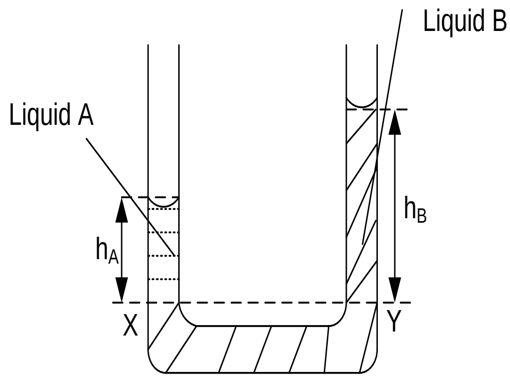
Pressure at C = (H-h)

Pressure at C = $760 - 30 = 730mmHg$

Gas pressure = $730mmHg$

11.6.3: DENSITY OF A LIQUID [U-TUBE MANOMETER]

It uses two immiscible liquids



The pressure P_x at X is equal to atmospheric pressure H plus the pressure exerted by the height h_A of liquid A i.e.

P_x = H+ $h_A \rho_A$g

Where ρ_A is the density of liquid A

Similarly at Y

P_y = H+ $h_B \rho_B$ g

Where ρ_B is the density of liquid B

Since x and y are at the same level

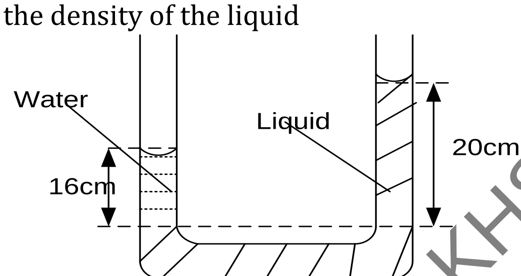
$P_x = P_y$

H+ $h_A \rho_A$g = H+ $h_B \rho_B$ g

$$\boxed{h_A\rho_A = h_B \rho_B}$$

Example

Find the density of the liquid



Solution

$$h_w\rho_w = h_L\rho_L$$

$$\frac{16}{100}x1000 = \frac{20}{100}x\rho_L$$

ρ_L = 800kgm^{-3}

UNEB 2014Q.4

(a) Define coefficient of viscosity and state its units (02marks)

(b) Explain the origin of viscosity in air and account for the effect of temperature on it (05marks)

(c) Describe, stating the necessary precautions an experiment to measure the coefficient of viscosity of a liquid using Stoke's law (07marks)

(d) A steel ball bearing of diameter 8.0mm falls steadily through oil and covers a vertical height of 20.0cm in 0.56 s. if the density of the steel is 7800kgm^{-3} and that of oil is 900 kgm^{-3}. Calculate:

(i) Up thrust on the ball **An $2.37x10^{-3}N$** (03 marks)

(ii) Viscosity of oil **An $0.674Nsm^{-2}$** (03 marks)

UNEB 2013 Q.2

(a) Define terminal velocity. (01mark)

(b) Explain laminar flow and turbulent flow. (03marks)

(c) Describe an experiment to measure the coefficient of viscosity of water using Poiseulle's formula. (07marks)

(d) (i) State Bernoulli's principle. (01marks)

(ii) Explain why a person standing near a railway line is sucked towards the railway line when a fast moving train passes. (03marks)

(e) A horizontal pipe of cross-sectional area 0.4 m^2, tapers to a cross-sectional area of 0.2 m^2. The pressure at the large section of the pipe is 8.0 x $10^4 N\,m^{-2}$ and the velocity of water through the pipe is 1.2 m s^{-1}. If atmospheric pressure is 1.01 x$10^5 N\,m^{-2}$, find the pressure at the small section of the pipe. **An[9.884x$10^4 N\,m^{-2}$,]** (05marks)

UNEB 2012 Q 4

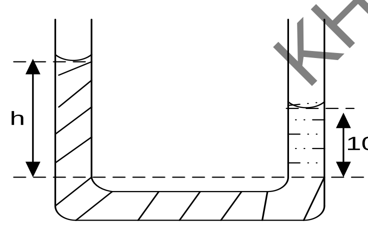
a) i) What is meant by the following terms steady flow and viscosity. (02marks)

ii) Explain the effect of increase in temperature on the viscosity of a liquid. (03marks)

b) i) Show that the pressure p exerted at a depth h below the free surface of a liquid of density ρ is given by $P = h\rho g$ (03marks)

ii) Define relative density (01mark)

iii) A U-tube whose ends are open to the atmosphere, contains water and oil as shown below.



Given that the density of oil is 800kgm^{-3}, find the value of h. **An [12.5cm]**

UNEB 2011 Q 3

a) i) What is meant by viscosity. (01mark)

ii) Explain the effect of temperature on the viscosity of a liquid. (03marks)

b) Derive an expression for the terminal velocity of a sphere of radius a, falling in liquid of viscosity η (4mks)

c) Explain why velocity of a liquid at a wide part of tube is less than that at a narrow part. (2mks)

UNEB 2010 Q 3

a) Define viscosity of a fluid (01mark)

b) i) Derive an expression for the terminal velocity attained by a sphere of density δ, and radius a, falling through a fluid of density ρ and viscosity η (05marks)

ii) Explain the variation of the viscosity of a liquid with temperature. (02marks)

UNEB 2009 Q 4

a) i) State Archimedes principle (01mark)

ii) A tube of uniform cross sectional area of $4x10^{-3}m^2$ and mass 0.2kg is separately floated vertical in water of density $1000kgm^{-3}$ and in oil of density $800kgm^{-3}$. Calculate the difference in the lengths immersed. **An [$1.25x10^{-2}m$]** (04marks)

UNEB 2006 Q 4

a) i)State Archimedes principle (01mark)

ii) Describe an experiment to determine relative density of an irregular solid which floats in water.

UNEB 2005 Q 3

a) What is meant by the following terms
   i) Velocity gradient (01mark)
   ii) Coefficient of viscosity (01mark)

b) Derive an expression for the terminal velocity of a steel-ball bearing of radius r and density ρ falling through a liquid of density σ and coefficient of viscosity η. (05marks)

d) Explain with the aid of a diagram why air flow cover the wings of an air craft at take-off causes a lift. (03marks)

UNEB 2003 Q 3

a) State the law of floatation . (01mark)

b) With the aid of a diagram describe how to measure the relative density of a liquid using Archimedes principle and the principle of moments. **An [refer to Abbot Pg 133]** (06marks)

c) A cross sectional area of a ferry at its water line is $720m^2$. If sixteen cars of average mass 1100kg are placed on board, to what extra depth will the boat sink in the water. **An[$2.4x10^{-2}m$]** (04marks)

UNEB 2002 Q 3

a) i) Show that the weight of fluid displaced by an object is equal to the up thrust on the object. (5mks)

ii) A piece of metal of mass $2.60x10^{-3}$kg and density $8.4x10^{3}kgm^{-3}$ is attached to a block of wax of mass $1.0x10^{-2}$ kg and density $9.2x10^{2}kgm^{-3}$. When the system is placed in a liquid, it floats with wax just submerged. Find the density of liquid. (04marks)

b) Explain the

i) Terms laminar flow and turbulent flow (04marks)

ii) Effects of temperature on the viscosity of liquids and gases (03marks)

c) i) Distinguish between static pressure and dynamic pressure (02marks)

Solution

a) ii) By law of floatation, a floating body displaces its own weight

Mass of liquid displaced = $(2.60x10^{-3} + 1.0x10^{-2})$

$= 1.26x10^{-2}$kg

Volume of liquid displaced = $\frac{2.6x10^{-3}}{8.4x10^{3}} + \frac{1x10^{-2}}{9.2x10^{2}}$.

$= 1.12x10^{5}m^{3}$

ρ of liquid $= \frac{mass\ of\ liquid\ displaced}{volume\ of\ liquid\ displaced}$

$= \frac{1.26\ x\ 10^{-2}}{1.12\ x\ 10^{-5}}$

ρ of liquid $= 1.13x10^{3}kg\ m^{-3}$

UNEB 1998 Q 4

a) i) Distinguish between laminar and turbulent flow (02marks)

ii) What are the origins of viscosity in liquid (02marks)

iii) Explain the temperature dependence of viscosity of a liquid. (02marks)

b) i) State Bernoulli's principle

ii) Account for the variation of pressure and velocity of a liquid flowing in a horizontal pipe of varying diameter. (04marks)

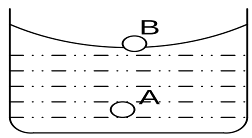
CHAPTER 12: SURFACE TENSION

The surface of a liquid behaves like an elastic skin in a state of tension.

It is responsible for the following observations;

1- A needle floating on an undisturbed water surface though made of material which is denser than water
2- Some insects walk on water surface without sinking
3- Drops of water remaining suspended and becoming nearly spherical when falling from a tap
4- Mercury gathering into small droplets when spilt

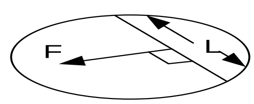
12.1.0: Molecular explanation for existence of surface tension



- Liquid molecules attract each other. In the bulk of the liquid the resultant force on any molecule such as A is zero.
- A surface molecule such as B is subjected to intermolecular forces of attraction below therefore potential energy of surface molecules exceeds that of the interior. Average separation of the surface molecules is greater than that of molecules in the interior. At any point on a liquid surface there is a net force away from that point and this makes the surface behave like an elastic skin in a state of tension. This accounts for surface tension.

Definition

Surface tension coefficient γof a liquid is defined as the force per unit length acting in the surface and perpendicular to one side of an imaging line drawn in the surface.



$$\gamma = \frac{F}{L}$$

Units of γ are Nm^{-1}

Dimensions of γ

$\gamma = \frac{F}{L}$

$[\gamma] = \frac{[F]}{[L]} = \frac{M\,L\,T^{-2}}{L}$

$[\gamma] = M\,T^{-2}$

Other units of γ are kgs^{-2}

12.1.2: Factors affecting surface tension

i) Temperature

When the temperature of a liquid is increased, the liquid molecules gain kinetic energy and the molecules become more free to move and rush to the surface. The number of molecules in the surface increase, potential energy of the surface molecules is lowered and the separation of molecules decreases leading to a reduction in the intermolecular attraction, this reduces tension energy of molecules and hence surface energy tension is also reduced.

ii) Impurities

Impurities detergents and soap get between the molecules of the liquid reducing the intermolecular forces between the liquids and hence reducing surface tension

iii) Nature of the liquid

Different liquids have different surface tension

12.1.3: SHAPES OF LIQUID SURFACE

The surface of a liquid must be at right angles to the resultant force acting on it otherwise there would be component of this force parallel to the surface which would cause motion.

Normally a liquid surface is horizontal i.e. at right angles with the force of gravity but where it's in contact with the solid it's usually curved.

The particular form that this curvature takes is determined by the strengths of what are called the **cohesive** and **adhesive** forces.

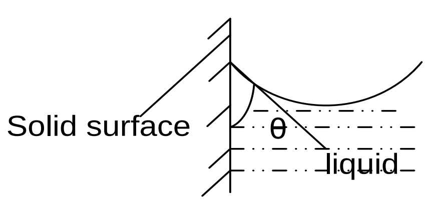
Definition

Cohesive force is the attractive force exerted on a liquid molecules by the neighboring liquid molecules.

Adhesive forces is the attractive force exerted on a liquid molecule by the molecules in the surface of the solid.

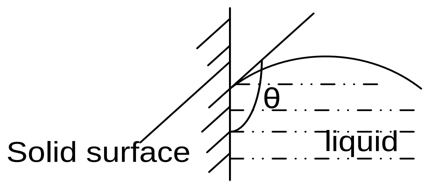
Consider a liquid in a container with vertical sides

- If the adhesive force is large comparative with the cohesive force, the liquid tends to stick to the wall and so has a concave meniscus (curves upwards).



e.g water and glass

- If the cohesive force is large compared with adhesive , the liquid surface pulls away from the wall and the meniscus is convex(curves downwards)



e.g mercury and glass

12.14: ANGLE OF CONTACT θ

This is the angle between the solid surface and the tangent plane to the liquid surface at the point where it touches the solid measured through the liquid.

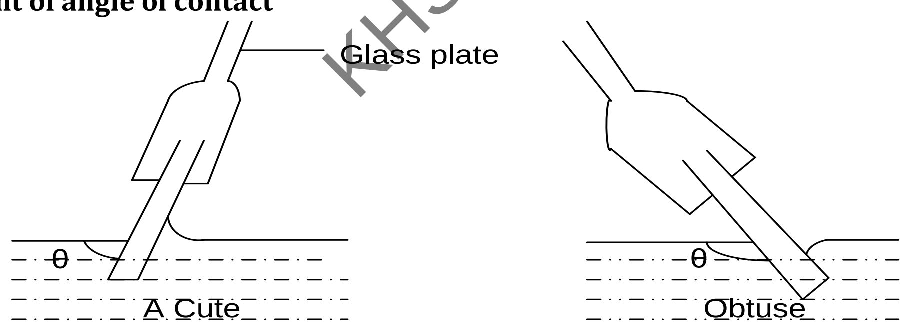
From the diagrams above, the meniscus is concave when θ is less than 90^0 and is convex when θ is greater than 90^0.

A liquid is said to wet a surface with which its angle of contact is less than 90^0.

The angle of contact of water and clean glass is **zero**, and that between mercury and clean glass is **137**°. Thus water wets clean glass, mercury does not.

Addition of a detergent to a liquid lowers its surface tension and reduces the contact angle.

Measurement of angle of contact

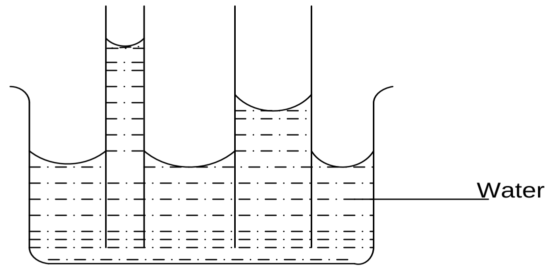


A clean glass plate is placed at varying angles to a liquid until the surface on one side of the plate remains horizontal. The angle θ made between the horizontal surface and the plate is the angle of contact.

12.3.0: CAPILLARITY

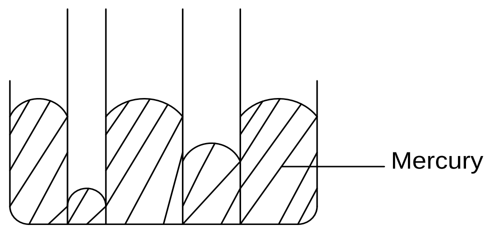
When a capillary tube is immersed in water and the plane vertical with one end of water. Water rises to a height above the surface of water in the container. This is due to the fact that adhesive forces are greater than the cohesive forces.

The narrower the tube, the greater is the height to which water rises.



If the capillary tube is dipped inside mercury liquid is depressed below the outside level. This is because the cohesion of mercury is greater than the adhesion of mercury and glass.

The depression of the tube increases with decreases the diameter of the tube



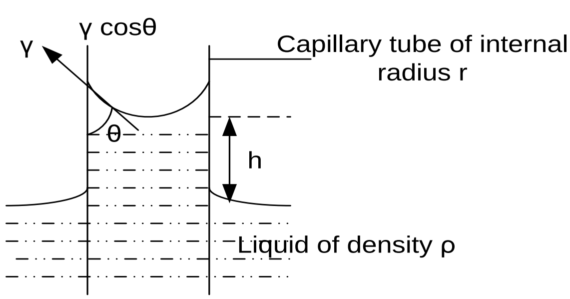
Definition

Capillarity: Is the rise or fall of a liquid in a capillary tube

12.3.1: Capillary rise

Around the boundary where the liquid surface meets the tube, surface tension forces exert a downward pull on the tube since they not balanced by any other surface tension forces.

The tube therefore exerts an equal but upwards force on the liquid which forces it to rise. The liquid stops rising when the weight of the raised column acting downwards equals to vertical component of the upward force exerted by the tube in the liquid.



Force acting upwards $F = \gamma \cos\theta \times L$

But $L = 2\pi r$

$F = \gamma \cos\theta \times 2\pi r$ ------------[1]

Weight $W = mg = V\rho g$

$W = Ah\rho g$

$W = \pi r^2 h\rho g$ ----------------- [2]

At equilibrium

Weight = vertical component of surface tension

$W = F$

$\pi r^2 h\rho g = \pi r$

$h = \frac{2\gamma \cos\theta}{r\rho g}$

12.3.2: Capillary depression

Consider mercury inside a tube and the angle of contact θ

$$P_2 - P_1 = \frac{2\,\gamma\,cos\theta}{r}$$

But P_1 = H(atmospheric)

$$P_2 - H = \frac{2\,\gamma\,cos\theta}{r}$$

$P_2 = \frac{2\,\gamma\,cos\theta}{r} + H$ ----------------------------- [1]

Also: $P_2 = H + h\rho g$------------------------- [2]

Equating

$$H + h\rho g = \frac{2\,\gamma\,\cos\theta}{r} + H$$

$$h\rho g = \frac{2\,\gamma\,cos\theta}{r}$$

$$\boxed{h = \frac{2\,\gamma\,cos\theta}{r\,\rho\,g}}$$

Example

1. A clean glass capillary tube of internal diameter 0.04cm is held with its lower end dipped in water contained in a beaker and with 12cm of the tube above the surface of water.
    i) To what height will water rise in the tube.
    ii) What will happen if the tube is now depressed until only 4cm of its length is above the surface.

(γ of water =7.0x10^{-2}Nm^{-1}, ρ of water =1000kgm^{-3})

Solution

i) Using $h = \frac{2\,\gamma\,\cos\theta}{r\,\rho\,g}$

But for a clean glass of water θ =0

$$h = \frac{2\,x\,7x\,10^{-2}\,cos0}{\left(\frac{0.04\,x10^{-2}}{2}\right)x\,1000\,x9.81}$$

h=0.071m

ii) If only 4cm of the tube is left above the water surface, this length is less than h in part (i) above so water must change its angle of contact so that it can fit the 4cm

$$h = \frac{2\,\gamma\,\cos\theta}{r\,\rho\,g}$$

$$4x10^2 = \frac{2\text{ x }7\text{x }10^{-2}\cos0}{\left(\frac{0.04\text{ x}10^{-2}}{2}\right)\text{x }1000\text{ x}9.81}$$

θ=55.9^0

water forms a new surface with an angle of contact 56^0

2. A U-tube is made with an internal diameter of one arm 2.0cm and the other 4mm and mercury is poured in the two tubes. If the angle of contact of mercury with glass after exposure to air is 160^0. What will be the difference in level of surface in the tubes, take surface tension of mercury as 0.0472Nm^{-1}

KHS

Solution

r_1=2x10^{-3}m r_2=1x10^{-2}m ρ =13600kgm^{-3}(density of mercury)

γ=0.0472   θ=180-160^0 θ=20^0(we subtracted to obtain a positive value of the $cos\theta$)

Note: we only subtract for angles greater than 90^0

$h_1 = \frac{2\,\gamma\,\cos\theta}{r_1\,\rho\,g} = \frac{2x0.047Cos20}{2x10^{-3}x13600x9.81}$ =3.32x10^{-4}m

$h_2 = \frac{2\,\gamma\,\cos\theta}{r_2\,\rho\,g} = \frac{2x0.047Cos20}{1x10^{-3}x13600x9.81}$ =6.65x10^{-5}m

Difference =h_1- h_2

$=3.32x10^{-4} - 6.65x10^{-5}$

$= 2.655x10^{-4}m$

Exercise: 32

1. A liquid of density 1000kg$m^{-3}$and surface tension $7.26x10^{-2}Nm^{-1}$, dipped in it is a capillary tube with a bore radius of 0.5mm. If the angle of contact is 0^0 determine,
   i) the height of the column of the liquid rise
   ii) if the tube is pushed until its 2cm above the level of the liquid, explain in what happen   **An[2.96x10^{-2}m, 47.5^0]**

2. The two vertical arms of manometer containing water, have different internal radii of 10^{-3}m and 2x10^{-3}m respectively. Determine the difference in height of the two liquids levels when the arms are open to the atmosphere. (surface tension and density of water are $7.2x10^{-2}Nm^{-1}$1 and 10^3kgm^{-3} respectively)   **An[7.14x10^{-3}m]**

3. The end of a clean glass capillary tube having internal diameter 0.6mm is dipped into a beaker containing water, which rises up the tube to a vertical height of 5.0cm above the water surface in the beaker. Calculate the surface tension of water (Density of water =1000kgm^{-3}, g=10ms^{-2}). What would be the difference if the tube were not perfectly clean so that the water did not wet it, but had an angle of contact of 30^0 with the tube surface.

   An[7.5x10Nm^{-1}, the water would rise to only 4.3cm]

4. A capillary tube which is clean is immersed in water of surface tension $7.2x10^{-2}Nm^{-1}$ and water rises 6.2cm in the capillary tube. What will be the difference in the mercury level, if the same capillary tube is immersed in the mercury (surface tension of mercury = 0.84 Nm^{-1}, angle of contact between mercury and glass =140^0, ρ of mercury =$1.36x10^4kgm^{-3}$, ρ of water =10^3kgm^{-3}3)

   An[h=4.2cm]

5. Mercury is poured into glass U-tube with vertical limbs of diameters 2.0mm and 12.0mm respectively. If the angle of contact between mercury and the glass is 140° and the surface tension of mercury is 0.52Nm^{-1}, calculate the difference in the levels of mercury. (density of mercury is $1.36x10^4 kgm^{-3}$) **An$(4.9x10^{-3}m)$**

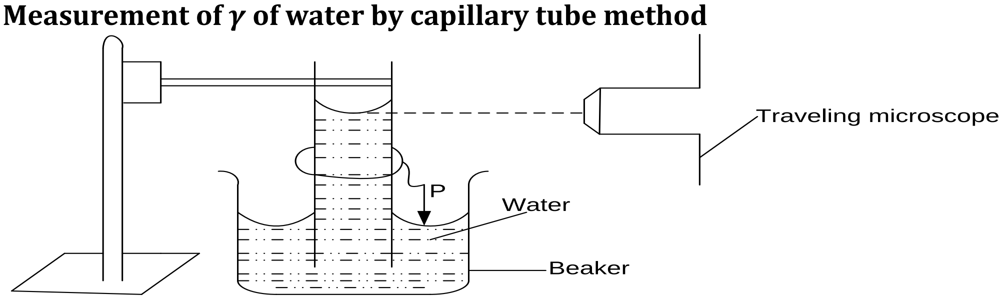
6. A U-tube with limbs of diameter 7mm and 4mmm contains water of surface tension $7x10^{-2}Nm^{-1}$, angle of contact 0° and density 1000kgm^{-3}. Find the difference in the levels. ***An* $\boldsymbol{3.1mm}$**

7. A glass U-tube is such that the diameter of one limb 4.0mm while that of the other is 8.0mm. the tube is inverted vertically with the open ends below the surface of water in a beaker. Given that surface tension of water is $7.2x10^{-2}Nm^{-1}$, angle of contact between water and glass is zero, and that density of water is 1000kgm^{-3}. What is the difference between the heights to which water rises in the two limbs. ***An* $\boldsymbol{7.34mm}$**

8. Calculate the height to which the liquid rises in the capillary tube of diameter 0.4mm placed vertically inside
   (i) A liquid of density $800kgm^{-3}$ and surface tension $5x10^{-2}Nm^{-1}$ and angle of contact 30°
   (ii) Mercury of angle of contact 139° and surface tension 0.52Nm^{-1}

   An[0.032m, 0.0294m]

12.4.O: Measurement of γ of water by capillary tube method



- ❖ A clean capillary tube is dipped in water as shown and a wire p which is bent is tied along the capillary tube with a rubber band.
- ❖ When the tube is dipped into water, the wire p is adjusted so that its top just touches the surface of the water.
- ❖ A travelling microscope is focused on the water meniscus in the capillary tube and the reading noted, say h_1.

- The beaker is then removes and the travelling microscope is focused on the tip of the wire p and scale reading h_2 is noted.
- The height of the water rise $h=/h_1-h_2/$.
- The capillary tube is removed and its diameter and hence radius , r is determined by using a travelling microscope. The surface tension can be obtained from ;

$$h=\frac{2\gamma\cos\theta}{r\rho g}$$

$$\gamma=\frac{h r\rho g}{2\cos\theta}$$

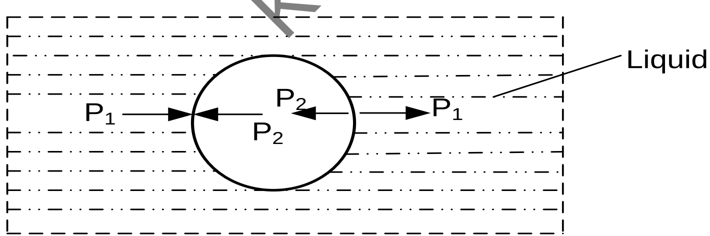
$\gamma=\frac{h r\rho g}{2}$ for clean glass of water $\theta=0^0$

12.2.0: PRESSURE DIFFERENCE ACROSS A SPHERICAL INTERFACE

The pressure inside a soap bubble is greater than the pressure of the air outside the bubble. If this were not so, the combined effect of the external pressure and the surface tension forces in the soap film would cause the bubble to collapse, similarly the pressure inside an air bubble in a liquid exceeds the pressure in the liquid and the pressure inside a mercury drop is greater than that outside it.

12.2.1: Pressure difference across an air bubble

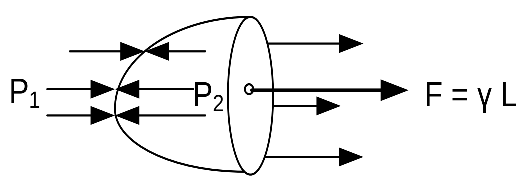
Consider an air bubble of radius r which is spherical and formed in a liquid of surface tension γ



P_1 = External pressure on the bulb due to the liquid

P_2 = internal pressure of air in the bubble

Considering half of the bubble. The remaining half experiences surface tension force due to the other half and this force acts towards the right.



For the bubble to maintain its shape the, internal pressure should be bigger than the external pressure.

At equilibrium; Force due to P_2 = force due to P_1 + surface tension

$$AP_2 = AP_1 + \gamma L$$

$$\pi r^2 P_2 = \pi r^2 P_1 + 2\pi r \gamma$$

$$\pi r^2 (P_2 - P_1) = 2\pi r \gamma$$

$$\boxed{P_2 - P_1 = \frac{2\gamma}{r}}$$

OR Excess pressure = $\frac{2\gamma}{r}$

Note:

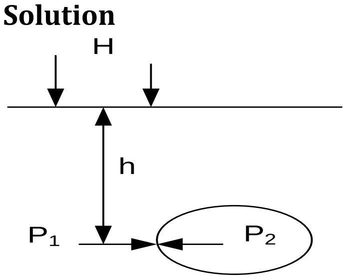
The pressure inside an air bubble is greater than that outside, otherwise the combined effect of the external pressure and the surface tension forces in the air bubble to collapse.

The same case can be extended to a soap bubble.

Example

Calculate the pressure inside a spherical air bubble of diameter 0.1cm blown at depth of 20cm below the surface of a liquid of density $1.26x10^3 kgm^{-3}$ and surface tension 0.064Nm^{-1}.(height of mercury barometer is 0.76m, and density of mercury is $13.6x10^3 kgm^{-3}$).

Solution



$P_1 = H + h\rho g$

$P_1 = 0.76x13.6x10^3 x9.81 + \frac{20}{100} x1.26x10^3 x9.81$

$P_1 = 101643$Pa

Excess pressure of air bubble = $\frac{2\gamma}{r}$

$P_2 - P_1 = \frac{2\gamma}{r}$

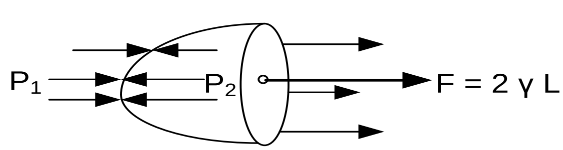
$P_2 - 101643 = \frac{2 \, x \, 0.064}{0.05 \, x \, 10^{-2}}$

$P_2 = 1.02x10^5$Pa

12.2.2: Excess pressure (pressure difference) for a soap bubble

For a soap bubble of radius r, there are two surfaces of liquid in contact with air (the air inside the bubble and air outside the bubble). Therefore the total length of surface in contact with air is 2L such that surface tension force.

$$F = 2L\gamma$$



At equilibrium : Inside force due to P_2 = external force due to P_1 +surface tension force

$$AP_2 = AP_1 + 2\gamma L$$

$$\pi r^2 P_2 = \pi r^2 P_1 + 4\pi r \gamma$$

$$\pi r^2 (P_2 - P_1) = 4\pi r \gamma$$

$$\boxed{P_2 - P_1 = \frac{4\gamma}{r}}$$

$$\text{Excess pressure} = \frac{4\gamma}{r}$$

Example

A soap bubble has a diameter of 4mm. calculate the pressure inside it, if the atmospheric pressure is $10^5 Nm^{-2}$, and that the surface tension of soap solution is $2.8x10^{-2} Nm^{-1}$

Solution

$P_2 - P_1 = \frac{4\gamma}{r}$

$P_2 - 10^5 = \frac{4 x 2.8x10^{-2}}{2 x 10^{-3}}$

$P_2 = 1.0006x10^5$Pa

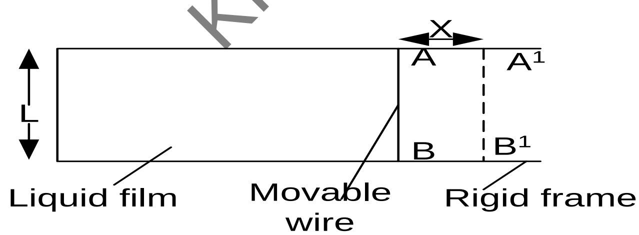
12.1.1: FREE SURFACE ENERGY (σ)

It is defined as the work done in creating a unit area of a new surface under isothermal conditions .

Units of σ are Jm^{-2} or Nm^{-1}

Consider stretching a thin film of a liquid on a horizontal frame as shown below. Since the film has both an upper and lower surface, the force F on AB due to surface tension is given by

$F = 2 L \gamma$------------------[1]



If AB is moved a distance x to A^1B^1, then work has to be done against this force

Work done = force x distance

$$W = Fx X = 2L \gamma X \text{----------------------------[2]}$$

The increases in surface area is 2LX (upper and lower surface).

Therefore the work done per unit area increases the surface energy (σ) is given by;

$$\sigma. = \frac{W}{2 L X} = \frac{2 L \gamma X}{2 L X}$$

$$\sigma = \gamma$$

∴ free surface energy =surface tension

Alternative definition of γ

Is the work done per unit area in increasing the surface area of a liquid under isothermal conditions.

Example

1. Calculate the work done against surface tension force on blowing a soap bubble of diameter 15mm , if the surface tension of the soap solution is $3.0x10^{-2}Nm^{-1}$.

Solution

$\gamma = \frac{work\ done}{increase\ in\ S.A}$

Work done $= \gamma\ x$ increase in surface area $= \gamma x(2x4\pi r^2)$

$=3.0x10^{-2}x2x4x\frac{22}{7}x\left(\frac{15x10^{-3}}{2}\right)^2$

Work done $= 4.241x10^{-5}$J

Increases in surface area is multiplied by 2 for both the upper and lower surface of a soap bubble.

2. Calculate the change in surface energy of a soap bubble when its radius decreases from 5cm to 1cm, given that the surface tension of soap solution is $2x10^{-2}Nm^{-1}$

Solution

$\gamma = \frac{work\ done}{increase\ in\ S.A}$

Work done $= \gamma\ x$ increase in surface area $= \gamma x(2x4\pi r^2)$

5cm bubble: Work done $=2.0x10^{-2}x2x4x\frac{22}{7}x(5x10^{-2})^2 = 1.257x10^{-3}$J

1cm bubble: Work done $=2.0x10^{-2}x2x4x\frac{22}{7}x(1x10^{-2})^2 = 5.027x10^{-5}$J

Change in surface energy **=**$1.257x10^{-3} - 5.027x10^{-5} = 1.207x10^{-3}$J

3. A liquid drop of diameter 0.5cm breaks up into 27 tiny droplets all of the same size. If the surface tension of the liquid is $0.07Nm^{-1}$ calculate the resulting change in energy.

Solution

Diameter of big drop, D= 0.5cm $\therefore R = 0.25cm = 2.5x10^{-3}m$

Volume of big drop= $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(2.5x10^{-3})^3$

Volume of 27 tiny droplets= $27x\frac{4}{3}\pi r^3$

$$27x\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2.5x10^{-3})^3$$

$\boldsymbol{r} = 8.3x10^{-4}m$

Work done $= \gamma\ x$ increase in surface area= $\gamma x(4\pi r^2)$

Big drop: Work done=$0.07x4x\frac{22}{7}x(2.5x10^{-3})^2 = 5.5x10^{-6}$J

27 drop lets: Work done= $27x0.07x4x\frac{22}{7}x(8.3x10^{-4})^2 = 1.637x10^{-5}$J

Change in surface energy **=**$1.637x10^{-5} - 5.5x10^{-6} = 1.087x10^{-5}$J

4. Calculate the work done in breaking up a drop of water of radius 0.5cm in to tiny droplets of water each of radius 1mm assuming isothermal conditions, given that surface tension of water is $7x10^{-2}$ Nm^{-1}.

Solution

Radius of big drop, R= 0.5cm $= 5x10^{-3}m$ and Radius of n tiny droplets, r= 1mm $= 1x10^{-3}m$

Volume of big drop= $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(5x10^{-3})^3$

Volume of n tiny droplets= $nx\frac{4}{3}\pi r^3 = nx\frac{4}{3}\pi(1x10^{-3})^3$

$$nx\frac{4}{3}\pi(1x10^{-3})^3 = \frac{4}{3}\pi(5x10^{-3})^3$$

$$\boldsymbol{n} = 125\ droplets$$

Work done = γ x increase in surface area= $\gamma x(4\pi r^2)$

Big drop: Work done= $7x10^{-2}\ x4x\frac{22}{7}x(5x10^{-3})^2 = 2.2x10^{-5}$J

125 drop lets: Work done= $125x7x10^{-2}\ x4x\frac{22}{7}x(1x10^{-3})^2 = 1.1x10^{-4}$J

Change in surface energy **=**$1.1x10^{-4} - 2.2x10^{-5} = x1.09x10^{-4}$J

EXERCISE: 33

1. A spherical drop of mercury of radius 2mm falls to the ground and breaks into 10smaller drops of equal size. Calculate the amount of work that has to be done.
   (Surface tension of mercury = 4.72 x 10^{-1} Nm^{-1}) **An[2.74x10^{-5}J]**

Relationship between surface area and shape of a drop

The area of a liquid has the least number of molecules in it under surface tensional forces. Surface are of a given volume of a liquid is there minimum when its is spherical and this explains why the meniscus and small droplets of mercury and rain are spherical in shape.

Why small mercury droplets are spherical and larger one flatten out

A small drop takes on a spherical shape to minimize the surface energy which tends to be greater than the gravitational potential energy. Therefore, the gravitational potential force cannot distort the spherical shape due to the very small mass of tiny droplets.

A large drop flattens out in order to minimize the gravitational potential energy, which tends to exceed the surface energy. Due to its large weight, gravitational force distorts the spherical shape of large drops. The shape of the drop must agree with the principle that the sum of gravitational potential energy and surface energy must be a minimum

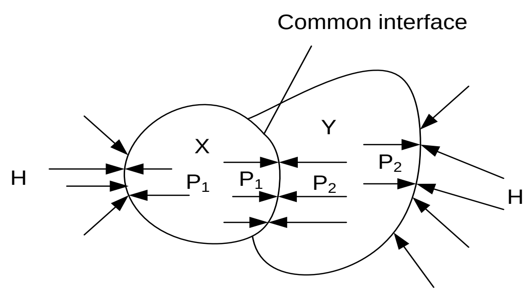
COMBINED BUBBLES

CASE 1

A soap bubble x of radius r_1, and another bubble y of radius r_2, are brought together so that the combined bubble has a common interface of radius R. show that

$$R=\frac{r_1\ r_2}{r_2-r_1}$$

Solution



Excess pressure on x

$P_1 - H = \frac{4\gamma}{r_1}$----------------------------- [1]

Excess pressure on y

$P_2 - H = \frac{4\gamma}{r_2}$ ----------------------------- [2]

Equation –equation 2 gives

$P_1 - P_2 = \frac{4\gamma}{r_1} - \frac{4\gamma}{r_2}$

$P_1 - P_2 = \frac{4\gamma}{r_1} - \frac{4\gamma}{r_2}$ ---------------------- [3]

Excess pressure at the interface

$P_1 - P_2 = \frac{4\gamma}{R}$ ----------------------------- [4]

Equating equation 3 and equation 4

$\frac{4\gamma}{R} = \frac{4\gamma}{r_1} - \frac{4\gamma}{r_2}$

$\frac{1}{R} = \frac{1}{r_1} - \frac{1}{r_2}$

$\frac{1}{R} = \frac{r_1 - r_2}{r_1 r_2}$

$R = \frac{r_1\ r_2}{r_2 - r_1}$

KHS

Example

1. A soap bubble x of radius 0.03m and another bubble y on radius,0.04m are brought together so that the combined bubble has a common interface of radius r. calculate r

Solution

Using $r = \frac{r_1\ r_2}{r_2 - r_1} = \frac{0.03x\ 0.04}{0.04 - 0.03}$ =0.12m

2. Two soaps bubble A and B of radii 6cm and 10cm respectively coalesce so that the combined bubble has a common interface . calculate the radius of curvature of this common surface and hence the pressure difference. Given that surface tension of soap is $2.5x10^{-2}Nm^{-1}$

Solution

Using $r = \frac{r_1 \ r_2}{r_2 - r_1} = \frac{0.06x\ 0.1}{0.1 - 0.06}$ =0.15m

pressure difference $= \frac{4\,\gamma}{r} = \frac{4x\ 2.5x10^{-2}}{0.15} = 0.667Pa$

CASE 2

Two bubbles of a soap solution of radii r_1 and r_2 of surface tension γ and pressure P coalence under isothermal conditions to form one bubble. Find the expression for the radius of the bubble formed.

Solution

Let R be the radius of the new bubble

A_1 be the surface area of bubble with radius r_1

A_2 be the surface area of bubble with radius r_2

A be the surface area of bubble with radius R

Under isothermal conditions, work done in enlarging the surface area of a bubble is given by

$$2\gamma A = 2\gamma A_1 + 2\gamma A_2$$

$$2\gamma 4\pi R^2 = 2\gamma 4\pi\, r_1{}^2. + 2\gamma 4\pi\, r_2{}^2$$

$$R^2 = r_1{}^2 + r_2{}^2$$

$$R = \sqrt{r_1{}^2 + r_2{}^2}$$

Examples

1. Two soap bubbles have radii of 3cm and 4cm, the bubbles are in a vacuum and they combine to form a single larger bubble. Calculate the radius of this bubble

Solution

$$R = \sqrt{r_1{}^2 + r_2{}^2} = \sqrt{3^2 + 4^2} = 5cm$$

2. Two soap bubbles of radii 2cm and 4cm respectively coalesce under isothermal conditions. If the surface tension of the soap solution is $2.5x10^{-2}$

Solution

$$R = \sqrt{r_1{}^2 + r_2{}^2} = \sqrt{(2x10^{-2})^2 + (4x10^{-2})^2} = \sqrt{20x10^{-4}}m$$

pressure difference $= \frac{4\,\gamma}{r} = \frac{4x\ 2x10^{-2}}{\sqrt{20x10^{-4}}} = 1.789Pa$

EXERCISE: 34

1. A soap bubble whose radius is 12mm becomes attracted to one of radius 20mm. Calculate the radius of curvature of the common interface. **An[30mm]**

2. Two soap bubbles of radii 2.0cm and 4.0cm respectively coalesce under isothermal conditions. If the surface tension of the soap solution is $2.5 \times 10^{-2} Nm^{-1}$. Calculate the excess pressure inside the resulting soap bubble. **An[2.36Pa]**

UNEB 2009 Q.4

C) i)Define surface tension in terms of work (1mk)

ii)Use the molecular theory to account for the surface tension of liquid (4mk)

iii)Explain the effect of increasing temperature of a liquid on its surface tension (4mk)

iv) Calculate the excess pressure inside a soap bubble of diameter 3.0cm if the surface tension of the soap solution is $2.5 \times 10^{-2} Nm^{-1}$. **An[6.67Pa]** (2mk)

UNEB 2008 Q.3

c) i) Define surface tension (01mark)

ii) Explain the origin of surface tension (03marks)

iii) Describe an experiment to measure the surface tension of a liquid by the capillary method (06marks)

UNEB 2002 Q.4

a) Define the term surface tension in terms of surface energy (01mark)

b) i) Calculate the work done against surface tension is blowing a soap bubble of diameter 15mm, if the surface tension of the soap solution is $3.0 \times 10^{-2} Nm^{-1}$ **An** [4.24×10^{-5}J] (03marks)

ii) A soap bubble of a radius r_1 is attached to another bubble of radius r_2. If r_1 is less than r_2. Show that the radius of curvature of the common interface is $\frac{r_1 \; r_2}{r_2 - r_1}$ (05marks)

UNEB 2001 Q.3

a) Define surface tension and derive its dimension (3mk)

b) Explain using the molecular theory the occurrence of surface tension (4mk)

c) Describe an experiment to measure surface tension of a liquid by the capillary tube method (6mk)

d) i)Show that the excess pressure in a soap bubble is given by $P = \frac{4\gamma}{r}$

ii) Calculate the total pressure within a bubble of air of radius 0.1mm in water, if the bubble is formed 10cm below the water surface and surface tension of water is $7.27 \times 10^{-2} Nm^{-1}$.

[Atmospheric pressure $=1.01 \times 10^{5}$Pa] **An 1.03×10^{5}Pa**

END